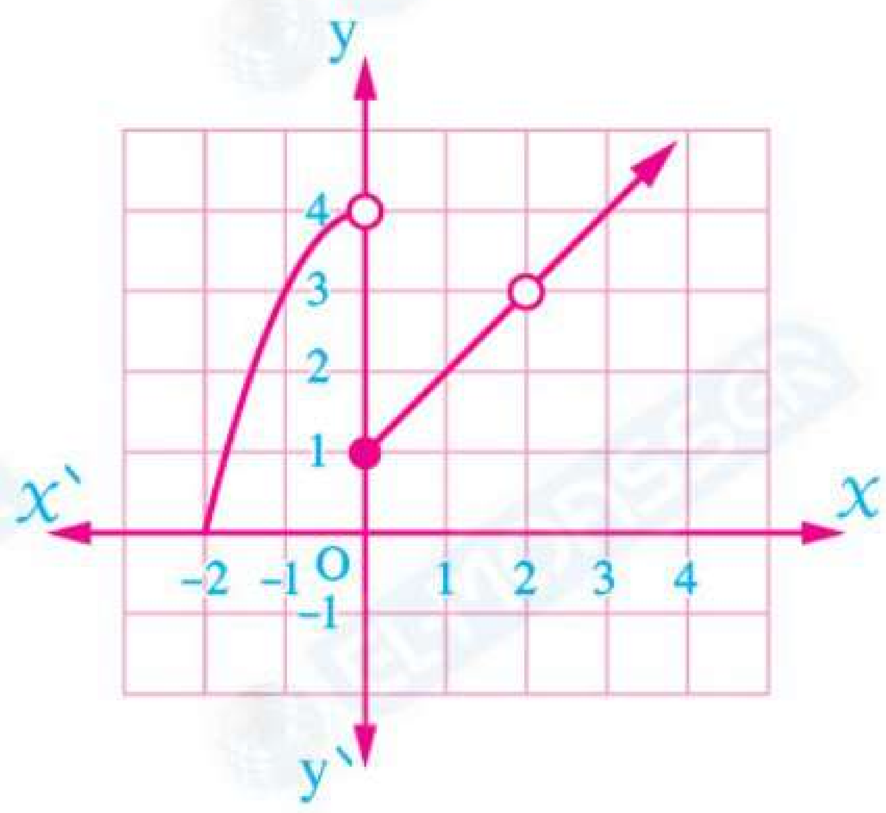
Test 1

Total mark

20

1 Choose the correct answer from those given : (12 marks)

1 **In the opposite figure :**



$\lim\limits_{x \to 0} f(x) =$ ....

(a) 2 (b) 4

(c) 1 (d) not exist.

2 In Δ ABC , $\frac{\sin A}{3} = \frac{2 \sin B}{5} = \frac{\sin C}{4}$, then a : b : c = ...

(a) 6 : 5 : 8 (b) 8 : 5 : 6 (c) 7 : 2 : 4 (d) 3 : 5 : 4

3 $\lim\limits_{x \to 3} \frac{x^2 - 9}{x^2 - 5x + 6} =$ ....

(a) 2 (b) 3 (c) 6 (d) 4

4 The function $f : f(x) = -x^2$ is decreasing when $x \in$ ....

(a) $\mathbb{R}$ (b) $\mathbb{R}^+$ (c) $\mathbb{R}^-$ (d) $\mathbb{R}^*$

5 In Δ ABC , if b = 5 cm. , m ($\angle$ B) = 30° , then the circumference of its circumcircle = ... cm.

(a) $50\sqrt{3}\,\pi$ (b) 5π (c) $10\sqrt{3}\,\pi$ (d) 10π

6 The curve of the function $f(x) = x^2 - 4$ is the same as the curve of the function $g(x) = x^2$ by translation 4 units in direction of ....

(a) $\overrightarrow{ox}$ (b) $\overrightarrow{ox'}$ (c) $\overrightarrow{oy'}$ (d) $\overrightarrow{oy}$

7 If the domain of the function $f : f(x) = \frac{1}{x^2 + kx + 9}$ is $\mathbb{R} - \{3\}$, then k = ...

(a) 6 (b) -6 (c) ± 6 (d) -36

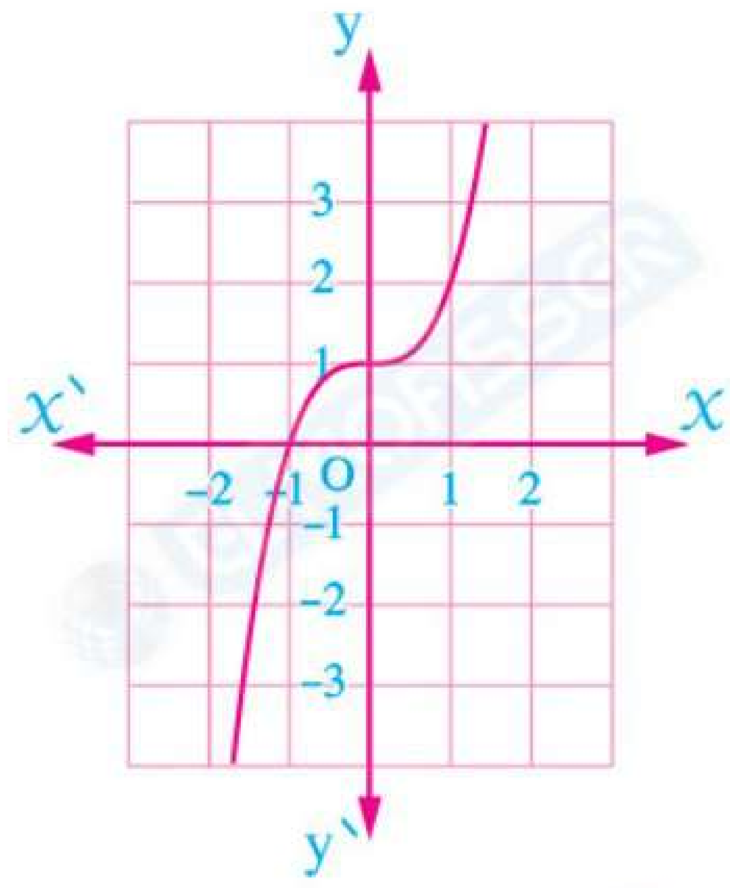
8 The range of the function $f : f(x) = \frac{1}{x} + 2$ is ....

(a) $\mathbb{R} - \{-3\}$ (b) $\mathbb{R} - \{2\}$ (c) $\mathbb{R} - \{3\}$ (d) $\mathbb{R} - \{2 , 3\}$

9 The type of the function $f : f(x) = x \sin x$ is ....

(a) even. (b) odd. (c) neither odd nor even. (d) constant.

10 The rule of the function represented by the opposite figure is ....



(a) $y = x^3 - 1$ (b) $y = (x + 1)^3$

(c) $y = (x - 1)^3$ (d) $y = x^3 + 1$

11 $\lim\limits_{x \to 1} \left(\frac{3}{4}\right) =$ ....

(a) 3 (b) 4 (c) $\frac{3}{4}$ (d) 1

12 $\lim\limits_{x \to 0} \frac{\sqrt{x+1} - 1}{x} =$ ....

(a) zero (b) $\sqrt{2}$ (c) $\frac{1}{2}$ (d) not exist.

2 Answer the following questions :

1 Graph the curve of the function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(x) = |x| - 4$, state range , type and the monotony. (2 marks)

2 Determine domain of the real function $f : f(x) = \frac{1}{\sqrt{3 - x}}$ (2 marks)

3 Find : $\lim\limits_{x \to 1} \frac{\sqrt{4x - 3} - 1}{x - 1}$ (2 marks)

4 Δ ABC in which m ($\angle$ B) = 35° , m ($\angle$ C) = 70° and radius of its circumcircle = 16 cm. Find area and perimeter of Δ ABC to nearest unit. (2 marks)

Test 2

Total mark
20

1 Choose the correct answer from those given : (12 marks)

1 The function $f : f(x) = 1 - |x|$ is increasing on .... where $f : \mathbb{R} \longrightarrow \mathbb{R}$

ⓐ $]1 , \infty[$ ⓑ $]0 , \infty[$ ⓒ $]-\infty , 1[$ ⓓ $]-\infty , 0[$

2 If $\lim\limits_{x \to 4} \frac{x^2 + 7x + a}{x^2 - 6x + 8} = \frac{15}{2}$, then a = ...

ⓐ -44 ⓑ 7 ⓒ -8 ⓓ 8

3 $\lim\limits_{x \to 0} \frac{(x+5)^2 - 25}{x} =$ ....

ⓐ 2 ⓑ 25 ⓒ 5 ⓓ 10

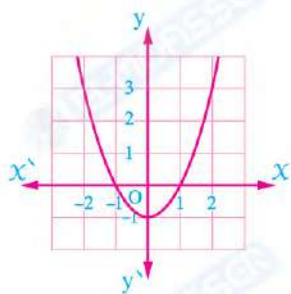
4 The point of symmetry of the function $f : f(x) = \frac{1}{x-2} + 1$ is ....

ⓐ $(-2 , 1)$ ⓑ $(-2 , -1)$ ⓒ $(2 , 1)$ ⓓ $(2 , -1)$

5 The domain of the function $f : f(x) = \begin{cases} 3 - x & , \quad -2 \le x < 2 \\ x & , \quad 2 \le x \le 5 \end{cases}$ is ....

ⓐ $[1 , 5]$ ⓑ $[-2 , 5]$ ⓒ $]1 , 5]$ ⓓ $[-2 , 2]$

6 Which of the following rules difined the curve of the function shown in the opposite figure ?



ⓐ $f(x) = x^3 - 1$ ⓑ $f(x) = x^2 - 1$

ⓒ $f(x) = x^3 + 1$ ⓓ $f(x) = x^2 + 1$

7 In Δ ABC , AB = 4 cm , Sin C = $\frac{1}{3}$, then the radius of the circle passes through its vertices = ... cm.

ⓐ 6 ⓑ 8 ⓒ 4 ⓓ 12

8 In Δ ABC , m ($\angle$ B) = 52° , m ($\angle$ C) = 48° , the perimeter of the triangle = 30 cm. , then a $\simeq$ .... (to the nearest cm.)

ⓐ 15 ⓑ 21 ⓒ 12 ⓓ 20

9 If f is an odd function , and its domain is $\mathbb{R}$, $a \in \mathbb{R}$, then $\frac{f(a) + f(-a)}{2} = \dots\dots$

ⓐ zero ⓑ $f(a)$ ⓒ $f(-a)$ ⓓ $f(0)$

10 Which of the following rules defined a function that is not odd ?

ⓐ $f(x) = \sin x$ ⓑ $f(x) = \sec x$ ⓒ $f(x) = x^3$ ⓓ $f(x) = \frac{1}{x}$

11 $\lim_{x \to -2} \frac{1}{|x|} = \dots\dots$

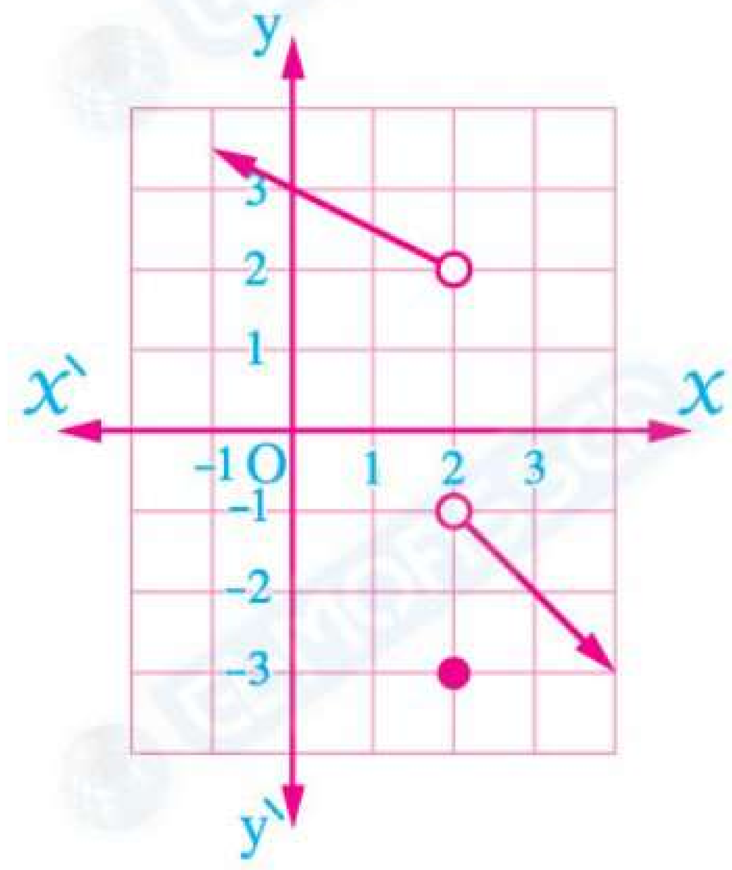
ⓐ 1 ⓑ -1 ⓒ $\frac{1}{2}$ ⓓ $-\frac{1}{2}$

12 In the opposite figure :

$\lim_{x \to 2} f(x) = \dots\dots$

ⓐ -3 ⓑ 2

ⓒ -1 ⓓ does not exist.



2 Answer the following questions :

1 Graph the function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(x) = (x-2)^3 - 1$, from the graph find the range , the monotony and determine if it is odd , even or otherwise. (2 marks)

2 If f_1 , f_2 are two real functions , $f_1(x) = x^5$, $f_2(x) = \sin x$

Determine the type of the function $(f_1 + f_2)$ where it is even , odd or otherwise. (2 marks)

3 **Find** : $\lim_{x \to -1} \frac{x+1}{\sqrt{x+5} - 2}$ (2 marks)

4 Δ ABC in which b = 10 cm. , m ($\angle$ A) = 40° , m ($\angle$ C) = 80°

, find the length of greatest side in the triangle (2 marks)

Answers of Test 1

1 1 (d) 2 (a) 3 (c) 4 (b) 5 (d) 6 (c)

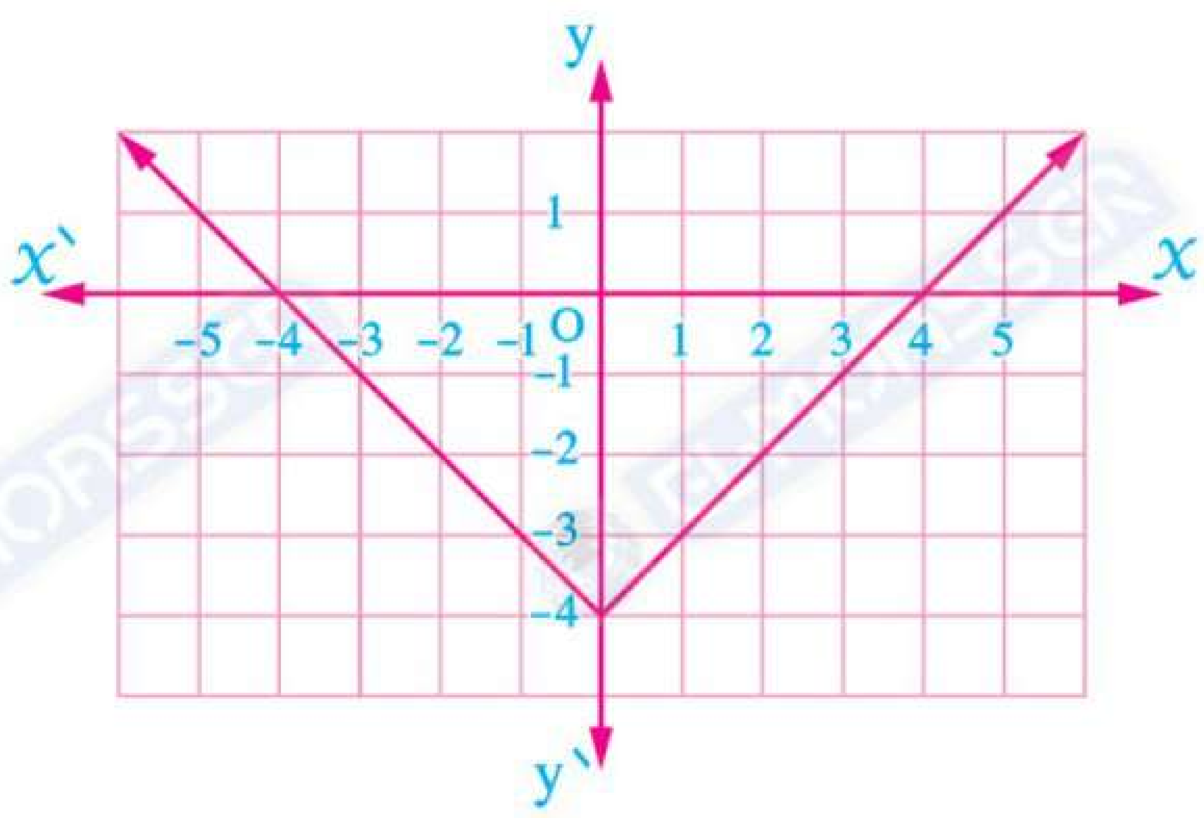
7 (b) 8 (b) 9 (a) 10 (d) 11 (c) 12 (c)

2 1 • The range = $[-4 , \infty[$

• The function is even

• The function is decreasing on $]-\infty , 0[$

and increasing on $]0 , \infty[$



2 Put $3 - x > 0$ $\therefore x < 3$

$\therefore x \in]-\infty , 3[$ $\therefore$ domain of $f =]-\infty , 3[$

3 $$\lim_{x \to 1} \frac{\sqrt{4x-3}-1}{x-1} \times \frac{\sqrt{4x-3}+1}{\sqrt{4x-3}+1} = \lim_{x \to 1} \frac{4x-3-1}{(x-1)\left(\sqrt{4x-3}+1\right)}$$

$$= \lim_{x \to 1} \frac{4\cancel{(x-1)}}{\cancel{(x-1)}\left(\sqrt{4x-3}+1\right)}$$

$$= \lim_{x \to 1} \frac{4}{\sqrt{4x-3}+1} = \frac{4}{1+1} = 2$$

4 $\because$ m ($\angle$ A) = 180° – (35° + 70°) = 75°

$$\therefore \frac{a}{\sin 75°} = \frac{b}{\sin 35°} = \frac{c}{\sin 70°} = 32$$

$\therefore$ a = 32 sin 75° $\simeq$ 30.9 cm.

, b = 32 sin 35° $\simeq$ 18.4 cm.

, c = 32 sin 70° $\simeq$ 30 cm.

$\therefore$ Area of the triangle = $\frac{1}{2} \times 30.9 \times 18.4 \times \sin 70° \simeq 267$ cm^2.

, perimeter of the triangle = 30.9 + 18.4 + 30 $\simeq$ 79 cm.

Answers of Test 2

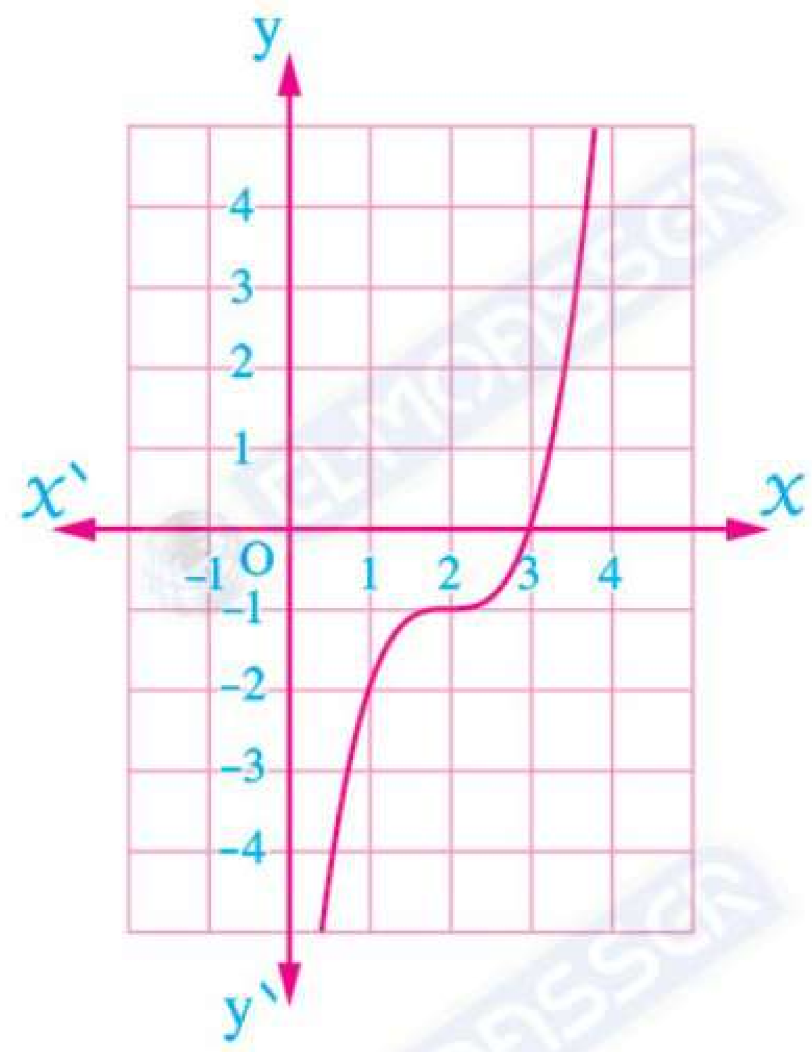
1

1 (d) 2 (a) 3 (d) 4 (c) 5 (b) 6 (b)

7 (a) 8 (c) 9 (a) 10 (b) 11 (c) 12 (d)

2

1
- The range = $\mathbb{R}$
- Increasing on $\mathbb{R}$
- The function neither odd nor even.



2 $\because f_1(-x) = (-x)^5 = -x^5 = -f_1(x)$

$\therefore f_1$ is odd function

, $\because f_2(-x) = \sin(-x) = -\sin x = -f_2(x)$

$\therefore f_2$ is odd function

$\therefore f_1 + f_2$ is odd function

3 $\lim\limits_{x \to -1} \frac{x+1}{\sqrt{x+5}-2} = \lim\limits_{x \to -1} \frac{(x+1)}{\sqrt{x+5}-2} \times \frac{\sqrt{x+5}+2}{\sqrt{x+5}+2}$

$= \lim\limits_{x \to -1} \frac{(x+1)\left(\sqrt{x+5}+2\right)}{x+5-4} = \lim\limits_{x \to -1} \left(\sqrt{x+5}+2\right) = 4$

4 $\because m(\angle B) = 180° - (40° + 80°) = 60°$

$\therefore \angle C$ is the greatest angle in measure

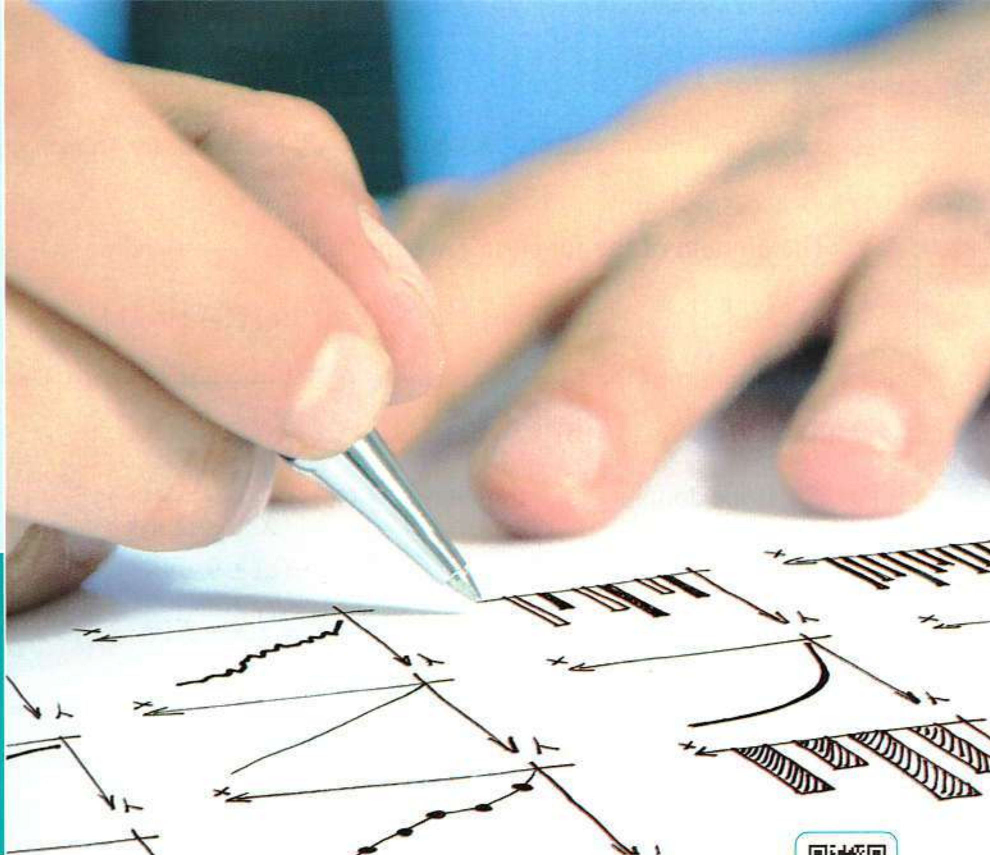
$\therefore$ c is the longest side

$\therefore \frac{c}{\sin 80°} = \frac{10}{\sin 60°}$

$\therefore c = \frac{10 \sin 80°}{\sin 60°} \simeq 11$ cm.

Exercise

1



Real functions

From the school book ● Understand ● Apply ● Higher Order Thinking Skills

Test yourself

First Multiple choice questions

Choose the correct answer from those given :

(1) In all the following relations , y is a function in x except ....

(a) $y = 3x + 1$ (b) $y = x^2 - 4$ (c) $x = y^2 - 2$ (d) $y = \sin x$

(2) In all the following relations , y is a function in x except ....

(a) $y = \cos x$ (b) $y = 2$ (c) $y = x^2 - 1$ (d) $y^2 = x^2 + 1$

(3) The domain of the function $f : f(x) = 5$ is ....

(a) $\mathbb{R}$ (b) $\mathbb{R}^+$ (c) $\{5\}$ (d) $\{0 , 5\}$

(4) The domain of the function $f : f(x) = \frac{2x + 1}{x - 2}$ is ....

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{-\frac{1}{2}\}$ (c) $\mathbb{R} - \{-\frac{1}{2} , 2\}$ (d) $\mathbb{R} - \{2\}$

(5) The domain of the function $f : f(x) = \frac{x + 5}{(x + 5)(x - 5)}$ is ....

(a) $\mathbb{R}$ (b) $\{5 , -5\}$ (c) $\mathbb{R} - \{5\}$ (d) $\mathbb{R} - \{5 , -5\}$

(6) The domain of the function $f : f(x) = \frac{x^2 + 1}{x^2 + 4x}$ is ....

(a) $\mathbb{R} - \{1 , -1\}$ (b) $\mathbb{R} - \{0 , -4\}$ (c) $\mathbb{R}$ (d) $\mathbb{R} - \{0 , 4\}$

(7) The domain of the function f where $f(x) = \frac{5 + 2x}{x^2 + x + 1}$ is ....

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{5\}$ (c) $\mathbb{R} - \{2\}$ (d) $\mathbb{R} - \{-2 , -5\}$

(8) The domain of the function f where $f(x) = \frac{7}{x^3 - x}$ is ....

(a) $\mathbb{R} - \{3\}$ (b) $\mathbb{R} - \{7\}$ (c) $\mathbb{R} - \{0, 1\}$ (d) $\mathbb{R} - \{0, 1, -1\}$

(9) The domain of the function f where $f : \mathbb{R}^+ \longrightarrow \mathbb{R}$, $f(x) = \frac{x-1}{4x}$ is ....

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{0\}$ (c) $\mathbb{R}^+$ (d) $\mathbb{R} - \{1\}$

(10) If the domain of the function $f : f(x) = \frac{2}{x^2 - 6x + k}$ is $\mathbb{R} - \{3\}$, then k = ...

(a) 3 (b) 9 (c) ± 9 (d) 18

(11) The domain of the function $f : f(x) = \sqrt{x - 3}$ is ....

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{3\}$ (c) $[3, \infty[$ (d) $]-\infty, 3[$

(12) The domain of the function f where $f(x) = \sqrt{4 - x}$ is ....

(a) $[4, \infty[$ (b) $]-\infty, 4[$ (c) $]4, \infty[$ (d) $]-\infty, 4]$

(13) The domain of the function $f : f(x) = \sqrt[3]{x - 5}$ is ....

(a) $[5, \infty[$ (b) $]-\infty, 5[$ (c) $\mathbb{R}$ (d) $\mathbb{R}^+$

(14) The domain of the function $f : f(x) = \sqrt[3]{9 - x^2}$ is ....

(a) $]-3, 3[$ (b) $\mathbb{R}$ (c) $\mathbb{R} -]-3, 3[$ (d) $[-3, 3]$

(15) The domain of the function $f : f(x) = \frac{5}{\sqrt{x - 4}}$ is ....

(a) $[4, \infty[$ (b) $]4, \infty[$ (c) $]-\infty, 4]$ (d) $]-\infty, 4[$

(16) The domain of the function f where $f(x) = \sqrt[4]{x^2 + 4}$ is ....

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{4\}$ (c) $\mathbb{R} - \{0\}$ (d) $\mathbb{R} - \{-2, 2\}$

(17) The domain of the function f where $f(x) = \frac{1}{\sqrt[3]{x^2 - 5x - 6}}$ is ....

(a) $\mathbb{R} - \{5\}$ (b) $\mathbb{R} - \{6\}$ (c) $\mathbb{R} - \{1, -6\}$ (d) $\mathbb{R} - \{-1, 6\}$

(18) If the domain of the function $f : f(x) = \frac{1}{\sqrt{x - a}}$ is $]-3, \infty[$, then a = ...

(a) 3 (b) -3 (c) ± 3 (d) $\sqrt{3}$

(19) If the domain of the function $f : f(x) = \frac{1}{\sqrt{x^2 + a}}$ is $\mathbb{R}$, then a can not be equal ....

(a) 5 (b) $\sqrt{4}$ (c) zero. (d) 9

(20) If : $f(x) = \begin{cases} -4x + 3 , & x < 3 \\ -x^3 , & 3 \le x \le 8 \\ 3x^2 + 1 , & x > 8 \end{cases}$, then $f(10) =$ ....

(a) -37 (b) -1000 (c) 301 (d) 43

(21) The domain of the function f where $f(X) = \begin{cases} -2 & , \quad X < 2 \\ 3 & , \quad X > 2 \end{cases}$ is ....

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{3\}$ (c) $\mathbb{R} - \{-2\}$ (d) $\mathbb{R} - \{2\}$

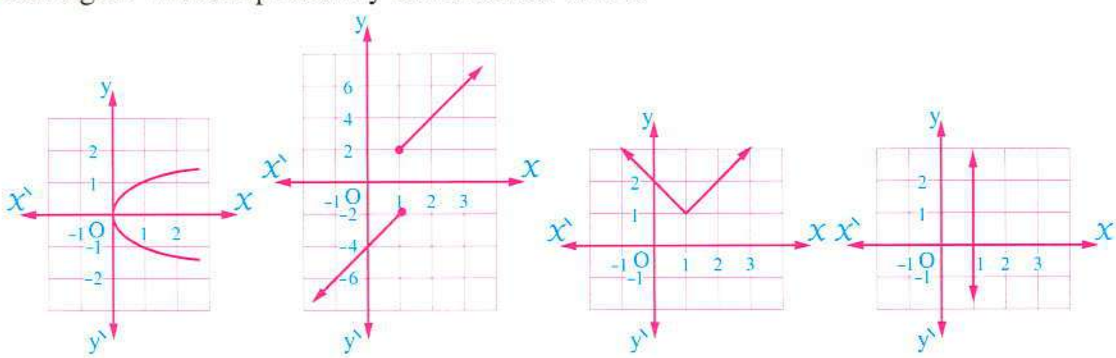
(22) The domain of the function f where $f(X) = \begin{cases} X & , \quad 0 \le X \le 1 \\ 2 - X & , \quad 1 < X \le 2 \end{cases}$ is ....

(a) $\mathbb{R} - \{1\}$ (b) $[0 , 2]$ (c) $\mathbb{R} - \{0 , 2\}$ (d) $]0 , 2[$

(23) The range of the function $f : f(X) = \begin{cases} 0 & , \quad X \le 0 \\ 1 & , \quad X > 0 \end{cases}$ is ....

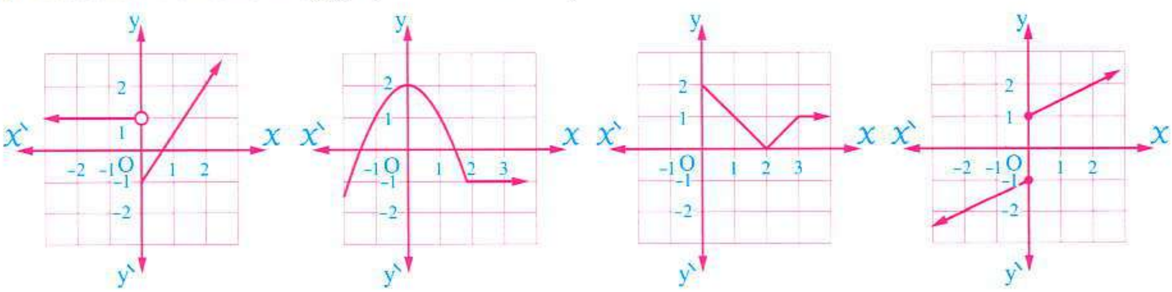
(a) $\{1\}$ (b) $\{0\}$ (c) $\mathbb{R}$ (d) $\{0 , 1\}$

(24) The figure which represents y as a function in X is ....



(a) (b) (c) (d)

(25) Which of the following graphs does not represent a function ?



(a) (b) (c) (d)

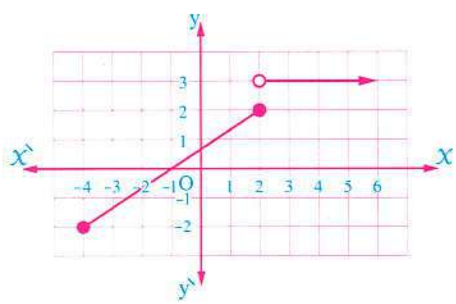
(26) The opposite figure represents ....

(a) function $f : [-4 , 2] \longrightarrow \mathbb{R}$

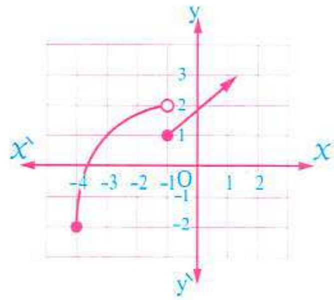
(b) function $f : [-4 , \infty[\longrightarrow \mathbb{R}$

(c) function $f : [-4 , 2] \longrightarrow [-2 , 3]$

(d) relation between X , y but not a function.



(27) The opposite figure represents the curve of function f , then its domain is ....



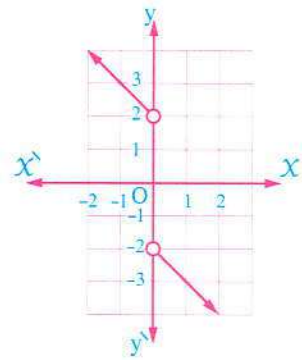
(a) $\mathbb{R} - \{-4 , -1\}$

(b) $]-4 , -1[$

(c) $[-4 , \infty[$

(d) $[-4 , \infty[- \{-1\}$

(28) The opposite figure represents function of x , its domain is ....



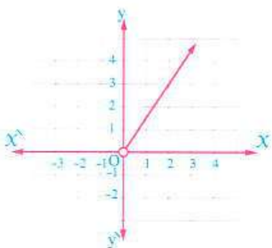
(a) $\mathbb{R}$

(b) $\mathbb{R} -]-2 , 2[$

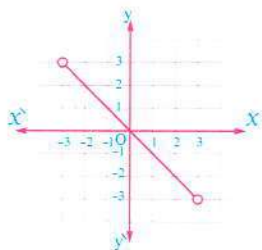
(c) $\mathbb{R} - [-2 , 2]$

(d) $\mathbb{R} - \{0\}$

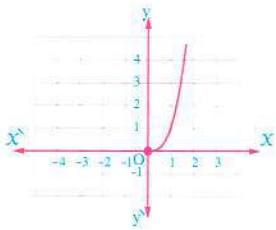
(29) Which of the following figures represents the curve of a function in which its range $\neq$ its domain ?



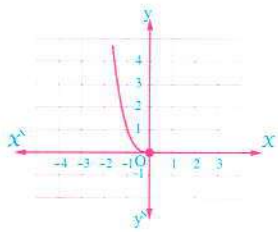
(a)



(b)

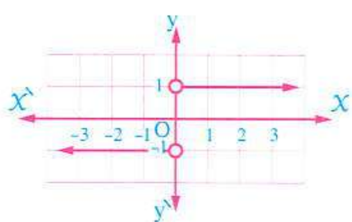


(c)



(d)

(30) The range of the function shown in the opposite figure is ....



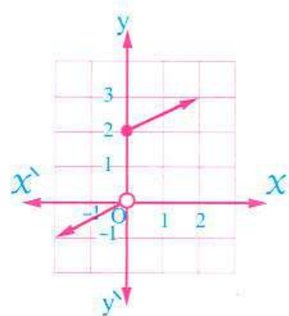
(a) $\{1\}$

(b) $\{1 , -1\}$

(c) $\{-1\}$

(d) $\mathbb{R} - \{0\}$

(31) The opposite figure represents a function of x , its range is ....



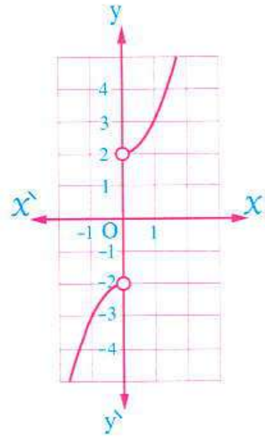
(a) $\mathbb{R} - [0 , 2]$

(b) $\mathbb{R} - \{0\}$

(c) $\mathbb{R} - [0 , 2[$

(d) $\mathbb{R} -]0 , 2]$

(32) In the opposite figure :



First : The range of the function is ....

(a) $\mathbb{R} - \{0\}$ (b) $\mathbb{R} - [-2 , 2]$

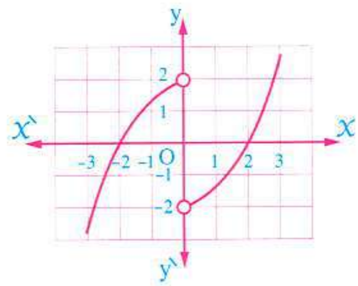
(c) $\mathbb{R}$ (d) $[-2 , 2]$

Second : The function is increasing in ....

(a) $]-\infty , 0[$ only (b) $]0 , \infty[$ only

(c) $]-\infty , 0[,]0 , \infty[$ (d) $\mathbb{R} - [-2 , 2]$

(33) In the opposite figure : If the drawn curve shows the function f , which of the following statements is true ?



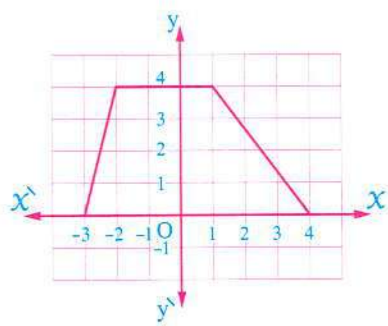
(a) The function increases on its domain.

(b) The function decreases on $]-\infty , -2[$ and increases on $]0 , \infty[$

(c) The function increases on each $]-\infty , 2[,]-2 , \infty[$

(d) The function increases on each $]-\infty , 0[,]0 , \infty[$

(34) The opposite figure represents the curve of the function f which of the following statements is false ?



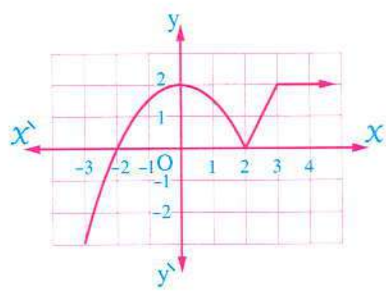
(a) f is constant on $]-2 , 1[$

(b) f is decreasing on $]1 , 4[$

(c) f is increasing on $]-3 , -2[$

(d) f is constant on $]-3 , 4[$

(35) In the opposite figure :



If the function decreases on $]0 , a[$ and constant on $]b , \infty[$, then a – b = ...

(a) 5 (b) 1

(c) – 1 (d) 3

Second Essay questions

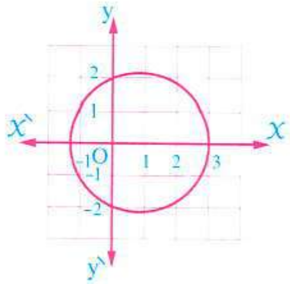
1 **If X and y are two real variables , then determine which of the following relations represents a function in X :**

(1) $y = 2X + 5$ (2) $y^2 = X + 4$ (3) $y = \sqrt{X^2 + 4}$

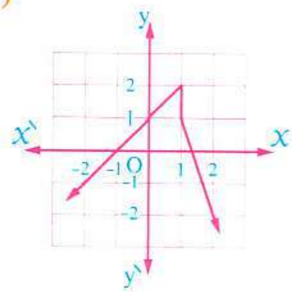
(4) $(X - y)^2 = 5$ (5) $y = 2$ (6) $X = 3$

2 **In each of the following graphs , show if y is a function in x or not :**

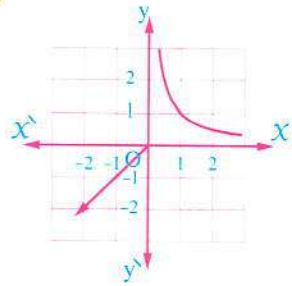
(1)



(2)



(3)



3 **Determine the domain of each of the real functions defined by the following rules :**

(1) $f(x) = \dfrac{2x+3}{x^2-3x+2}$

(2) $f(x) = \dfrac{8}{x^2-6x+9}$

(3) $f(x) = \dfrac{x+3}{3x^2-x-2}$

(4) $f(x) = \dfrac{x+1}{x^3+1}$

4 **Determine the domain of each of the real functions defined by the following rules :**

(1) $f(x) = \sqrt{x}$

(2) $f(x) = \dfrac{4}{\sqrt[3]{2x-5}}$

(3) $f(x) = \dfrac{5}{\sqrt{x+4}}$

(4) $f(x) = \dfrac{1}{\sqrt{3-x}}$

5 **Determine the domain of each of the real functions defined by the following rules :**

(1) $f(x) = \begin{cases} -3 & , \quad x < 3 \\ 5-x & , \quad x \ge 3 \end{cases}$

(2) $f(x) = \begin{cases} x^2-1 & , \quad x \le 2 \\ -5 & , \quad 2 < x < 4 \end{cases}$

(3) $f(x) = \begin{cases} 3x & , \quad x \in [0 , 2] \\ 6 & , \quad x \in]2 , 4[\\ x+2 & , \quad x \in [4 , 6] \end{cases}$

6 If $f : X \longrightarrow \mathbb{R}$ and $X = \{1 , 2 , -2 , -3\}$

, find the range of the function if $f(x) = 5x - 3$

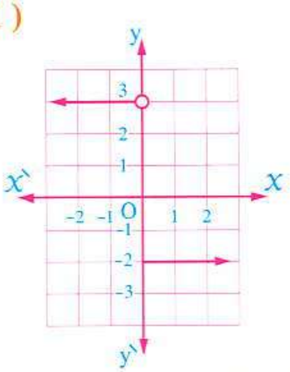
7 If $g : \{1 , 2 , 3 , 4 , 5\} \longrightarrow \mathbb{Z}^+$ where $g(x) = 4x - 3$

(1) Write down the range of the function.

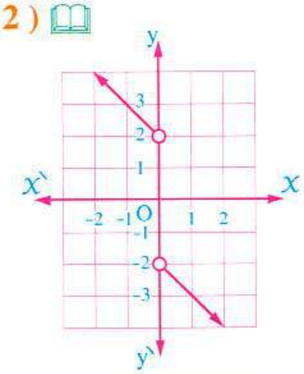
(2) If g (k) = 17 , find the value of k

8 **Determine the domain and range , then discuss the monotony of each of the functions represented by the following graphs :**

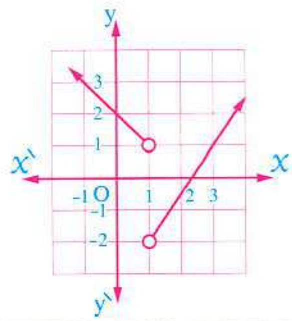
(1)



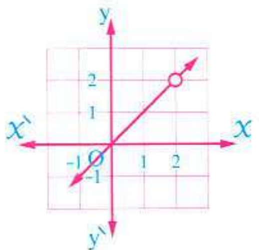
(2)



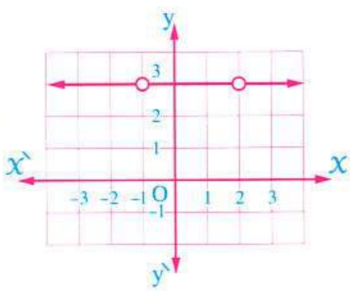
(3)



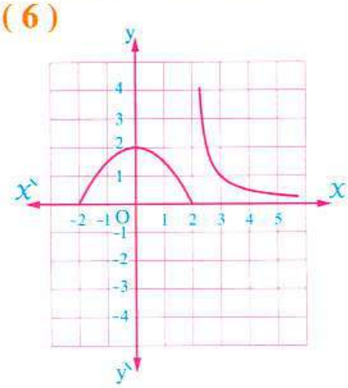
(4)



(5)



(6)



Third — Higher skills

Choose the correct answer from those given :

(1) If the relation between the sum of the interior angle measures of a polygon (y) and the number of its sides (x) is $y = \pi (x - 2)$, then the domain of this function is ……………

(a) $\mathbb{R}^+$ (b) $\mathbb{R} - \{2\}$ (c) $\mathbb{Z}^+$ (d) $\mathbb{Z}^+ - \{1 , 2\}$

(2) The domain of the function $f : f(x) = \frac{x}{\sqrt[3]{x} - 2}$ is ……………

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{2\}$ (c) $\mathbb{R} - \{0 , 2\}$ (d) $\mathbb{R} - \{8\}$

(3) The domain of the function $f : f(x) = \frac{x}{\sqrt{3x} - x}$ is ……………

(a) $]0 , \infty[$ (b) $]-\infty , 0[$ (c) $[0 , \infty[- \{1\}$ (d) $]0 , \infty[- \{3\}$

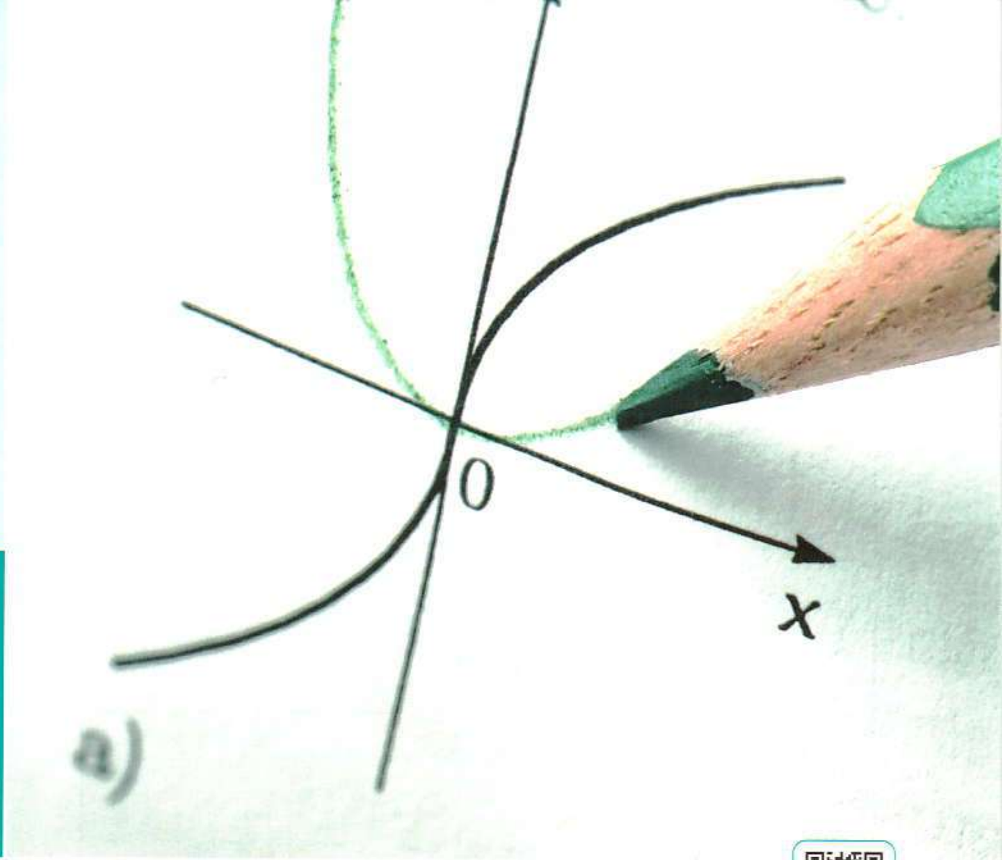
(4) The domain of the function $f : f(x) = \frac{5}{\sqrt{x-1} - 3}$ is ……………

(a) $[1 , \infty[$ (b) $[1 , \infty[- \{3\}$ (c) $[1 , \infty[- \{10\}$ (d) $[-3 , \infty[$

Exercise

2

Even and odd functions



From the school book ● Understand ● Apply ● Higher Order Thinking Skills

Test yourself

First Multiple choice questions

Choose the correct answer from those given :

(1) The even function from the functions that are defined by the following rules is ....

(a) $f(x) = x^3$ (b) $f(x) = \sin x$

(c) $f(x) = x \cos x$ (d) $f(x) = x \sin x$

(2) The odd function from the functions that are defined by the following rules is ....

(a) $f(x) = x^2 \sin x$ (b) $f(x) = \tan^2 x$

(c) $f(x) = \cos x$ (d) $f(x) = 1$

(3) The type of the function $f : f(x) = \frac{\sin x}{x}$ is ....

(a) even. (b) odd.

(c) neither even nor odd. (d) linear.

(4) The function $f : f(x) = x \cos x$ is ....

(a) even. (b) odd.

(c) neither even nor odd. (d) linear.

(5) The following rules of functions are even except ....

(a) $f(x) = \sin x$ (b) $f(x) = \cos x$ (c) $f(x) = x^2 - 1$ (d) $f(x) = 1$

(6) Which of the following rules is not even function ?

(a) $y = \frac{1}{x^2}$ (b) $y = \sec x$ (c) $y = x^2 + \sin x$ (d) $y = 3x^4 + 27$

(7) If $f(X) = \frac{1}{\sin X}$, then ....

(a) $f(X) = \frac{1}{f(X)}$ (b) $f(X) = -f(-X)$ (c) $f(X) = f(-X)$ (d) $f(-X) = f\left(\frac{1}{X}\right)$

(8) If f is an odd function , $f(1) = 2$, then which of the following points lies on the curve of f ?

(a) $(-1, 2)$ (b) $(-1, -2)$ (c) $(1, -2)$ (d) $(-1, 0)$

(9) If f is an odd function , $a \in$ the domain of f , then $f(a) + f(-a) =$ ....

(a) zero (b) $2f(a)$ (c) $2a$ (d) $f(a)$

(10) If f is an odd function , then $f(a) - f(-a) =$ ....

(a) zero. (b) $f(a)$ (c) $2f(a)$ (d) $f(2a)$

(11) If f is an even function , then $f(a) - f(-a) =$ ....

(a) zero. (b) $f(a)$ (c) $2f(a)$ (d) $f(2a)$

(12) If f is an even function , $2 \in$ the domain of f , then $f(2) + f(-2) =$ ....

(a) zero. (b) 4 (c) 2 (d) $2f(2)$

(13) If the function f is an even over $[a, b]$, then $b =$ ....

(a) a (b) $-a$ (c) $2a$ (d) a^3

(14) If f is a function where $f :]-5, 5] \longrightarrow \mathbb{R}$, $f(X) = X^2$, then the function f is ....

(a) even. (b) odd.

(c) linear. (d) neither odd nor even.

(15) If $f : f(X) = aX^3 + bX + c$ is an odd function , then $c =$ ....

(a) 2 (b) 1 (c) zero. (d) -1

(16) If $f : f(X) = X^2 + aX + 9$ is an even function , then $a =$ ....

(a) 6 (b) 3 (c) zero. (d) -6

(17) If $f(X) = X^3 - X$, then $|f(X) + f(-X)| =$ ....

(a) zero. (b) 1 (c) 2 (d) 4

(18) If $f : f(X) = aX^3 + b$ is an odd function and the curve of the function passes through the point $(2, 8)$, then $a + b^2 =$ ....

(a) zero. (b) -1 (c) 1 (d) 5

(19) The function $f : f(X) = X^3 + 5X$ is symmetric about ....

(a) the X-axis. (b) the y-axis.

(c) the origin. (d) can not be determined.

(20) The function $f : f(X) = X^2 + X^4 + 1$ is symmetric about ....

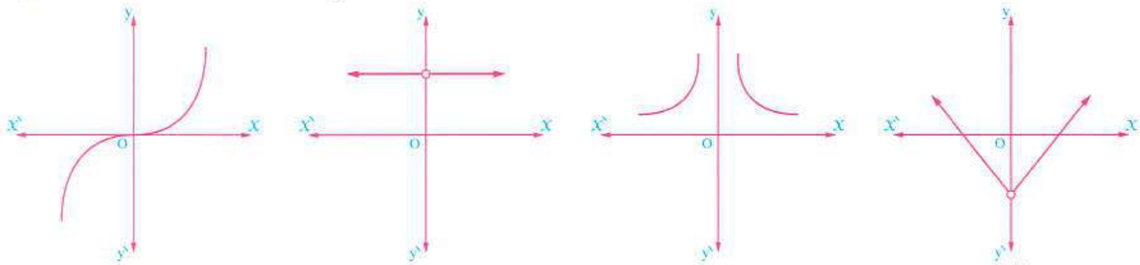
(a) the origin. (b) the X-axis.

(c) the y-axis. (d) it has neither symmetric point nor symmetric line.

(21) The function $f : f(X) = \sin 3X$ is symmetric about the point ....

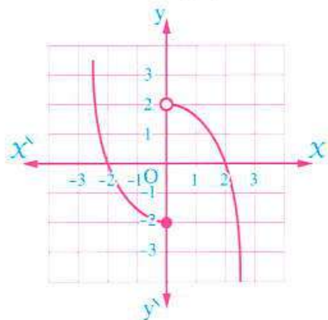
(a) (0 , 0) (b) (3 , 0) (c) (– 3 , 0) (d) (– 3 , 3)

(22) Which of the following functions is not even ?



(a) (b) (c) (d)

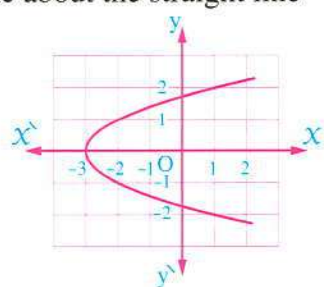
(23) The opposite figure represents the curve of the function f , then f is ....



(a) linear. (b) an even function.

(c) an odd function. (d) neither odd nor even.

(24) The curve represented in the opposite figure is symmetric about the straight line whose equation is ....



(a) $X = 0$ (b) $y = 0$

(c) $y = -2$ (d) $X = 2$

Second Essay questions

1 **In each of the following figures , mention the curve which is symmetric about the X-axis , the y-axis or the origin point :**

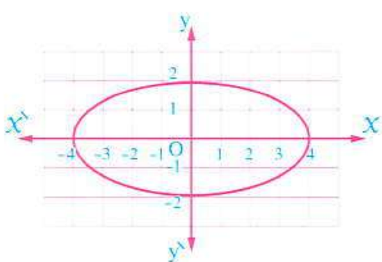


Fig. (1)

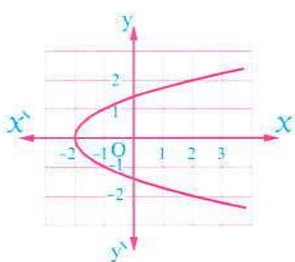


Fig. (2)

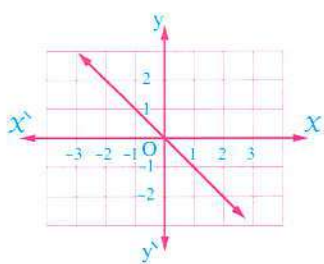


Fig. (3)

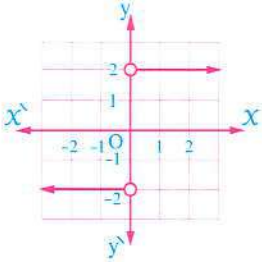


Fig. (4)

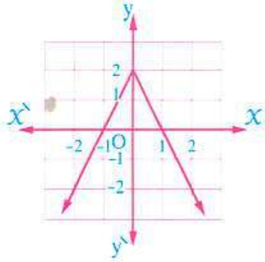


Fig. (5)

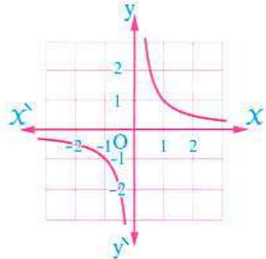
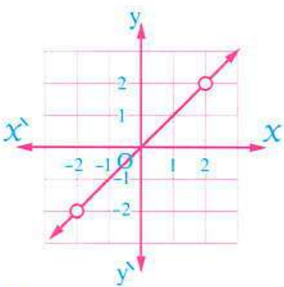


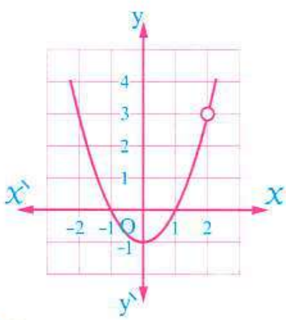
Fig. (6)

2 **Determine which of the functions represented by the following graphs is even , odd or neither even nor odd :**

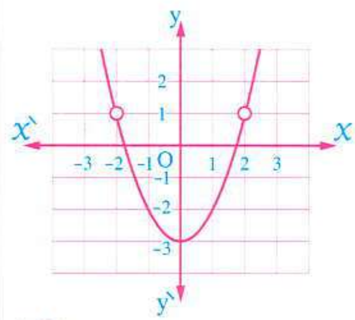
(1)



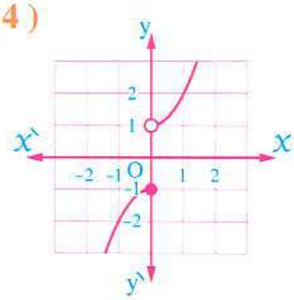
(2)



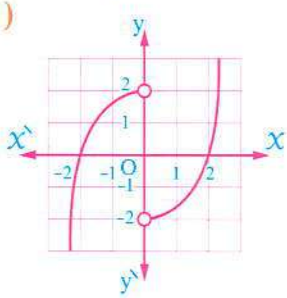
(3)



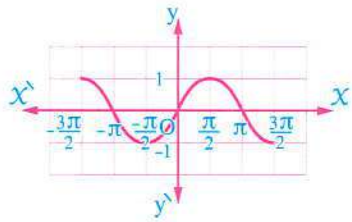
(4)



(5)



(6)



3 **Each of the following graphs represents the curve of the function f , determine whether the function f is even , odd or otherwise verifying your answers algebraically :**

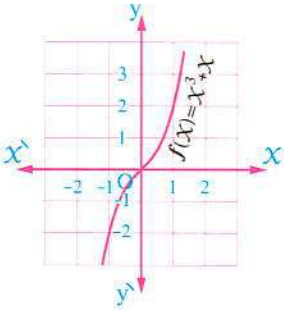


Fig. (1)

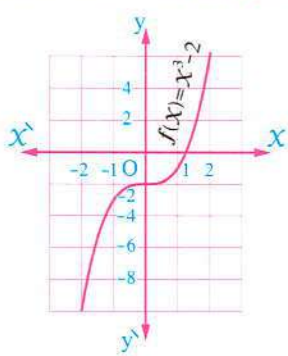


Fig. (2)

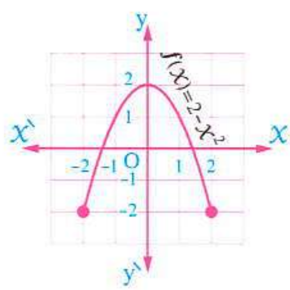


Fig. (3)

4 **Use the following figures to answer the following questions :**

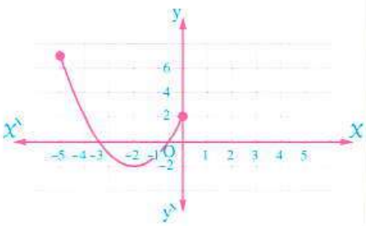


Fig. (1)

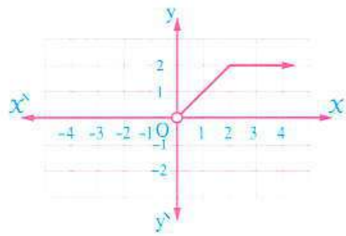


Fig. (2)

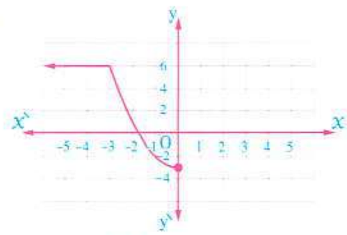


Fig. (3)

First : Complete the curve in each of fig. (1) and fig. (3) in your notebook to get an even function over its domain.

Second : Complete the curve in each of fig. (2) in your notebook to get an odd function over its domain.

Third : Determine the domain and the range of the function in each case , then investigate its monotony.

---

5 **Determine which of the functions defined by the following rules is even , which is odd and which is neither even nor odd :**

(1) $f(x) = 5$

(2) $f(x) = x^4 + x^2 - 1$

(3) $f(x) = 3x - 4x^3$

(4) $f(x) = x^2 - 3x + 4$

(5) $f(x) = x^3(x^2 - 1)$

(6) $f(x) = (x - 3)^2 - 7$

(7) $f(x) = \frac{x^3 + 2}{x - 3}$

(8) $f(x) = \frac{2x^3 - x^5}{x}$

(9) $f(x) = \sqrt{x + 3}$

(10) $f(x) = \sqrt[3]{x^3 + x}$

(11) $f(x) = x^3 - \frac{1}{x}$

(12) $f(x) = \left(x - \frac{2}{x}\right)^3$

(13) $f(x) = x \cos x$

(14) $f(x) = \frac{3x}{\tan x}$

(15) $f(x) = \frac{x^3 \sin 3x}{1 + x^4}$

(16) $f(x) = x^2 \sin^3 x$

(17) $f(x) = x \sin x^3$

(18) $f(x) = \frac{x^2 + \tan x}{x^4 + \sin x}$

6 If f_1 , f_2 and f_3 are three real functions where $f_1(X) = X^5$, $f_2(X) = \sin X$ and $f_3(X) = 5X^2$, tell which of the following functions is even , odd or otherwise :

(1) $f_1 + f_2$ (2) $f_1 + f_3$ (3) $f_1 \times f_2$ (4) $f_3 \times f_2$

---

7 Let f_1 , f_2 , g_1 and g_2 be real functions such that :

$f_1(X) = X^4$, $f_2(X) = \cos^5 X$, $g_1(X) = 2X^3$ and $g_2(X) = \sin^3 X$

Determine which of the following functions is even , odd or otherwise :

(1) $f_1 + g_2$ (2) $f_1 - f_2$ (3) $g_1 + g_2$

(4) $f_1 \times g_2$ (5) $g_1 \times g_2$ (6) $\frac{f_2}{f_1}$

---

8 Determine which of the following functions is even , odd or otherwise :

(1) $f : \mathbb{R} \longrightarrow \mathbb{R}$, $f(X) = X + 2$

(2) $f(X) = X^2$, $f : \mathbb{Z}^+ \longrightarrow \mathbb{Z}$

(3) $f : [-3 , 3[\longrightarrow \mathbb{R}$, $f(X) = 3X^2$

(4) $f(X) = X^2$, $f : \mathbb{R}^+ \longrightarrow \mathbb{R}^+$

(5) $f : f(X) = X^2$, $X \in \mathbb{R} - \{3\}$

Third Higher skills

Choose the correct answer from those given :

(1) If f is an odd function whose domain is $\mathbb{R}$, then $\frac{7f(-5) + 3f(5)}{2f(-5)} =$ ....

(a) 5 (b) – 5 (c) 2 (d) – 2

(2) If f is an even function whose domain is $\mathbb{R}$, then $\frac{7f(-5) + 3f(5)}{2f(-5)} =$ ....

(a) 5 (b) – 5 (c) 2 (d) – 2

(3) If f is an even function and $f(X) + X^2 f(-X) = 3$, then $f(1) =$ ....

(a) $\frac{1}{4}$ (b) 1 (c) $1\frac{1}{2}$ (d) 2

(4) If f is an odd function and $f(1) = k$ and $f(X + 2) = f(X) + f(2)$, then $f(3) =$ ....

(a) zero (b) 3 k (c) 6 k (d) 9 k

Exercise

3

Graphical representation of basic functions and graphing piecewise functions

From the school book · Understand · Apply · Higher Order Thinking Skills

Test yourself

First Multiple choice questions

Choose the correct answer from those given :

(1) If $f(x) = 5$, then the domain of the function f is ....

(a) $\mathbb{R}$ (b) $\mathbb{R}^+$ (c) $\{5\}$ (d) $\mathbb{R} - \{5\}$

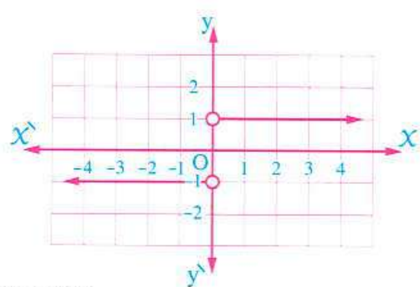
(2) If $f(x) = 7$, then the range of the function f is ....

(a) $\mathbb{R}$ (b) $\mathbb{R}^+$ (c) $\{7\}$ (d) $\mathbb{R} - \{7\}$

(3) The range of the function $f : f(x) = \begin{cases} 0 & \text{when } x \leq 0 \\ 1 & \text{when } x > 0 \end{cases}$ is ....

(a) $\{1\}$ (b) $\{0\}$ (c) $\mathbb{R}$ (d) $\{0 , 1\}$

(4) **In the opposite figure :**



The range of the function is ....

(a) $\{1\}$ (b) $\{1 , -1\}$

(c) $\{-1\}$ (d) $\mathbb{R}$

(5) The range of the function $f : f(x) = \begin{cases} x & , \quad x > 0 \\ -2 & , \quad x \leq 0 \end{cases}$ is ....

(a) $\mathbb{R}^+$ (b) $\mathbb{R}^+ - \{-2\}$ (c) $\mathbb{R}^+ \cup \{-2\}$ (d) $\mathbb{R}$

(6) The function f where $f(x) = \begin{cases} 2 & , \quad x > 0 \\ -2 & , \quad x < 0 \end{cases}$ is symmetric about the point ....

(a) $(2, 0)$ (b) $(-2, 0)$ (c) $(0, 0)$ (d) $(2, -2)$

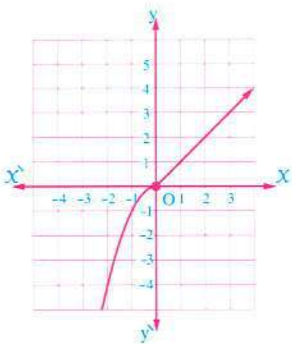
(7) The axis of symmetry for the function $f : f(x) = x^2$ is the straight line ....

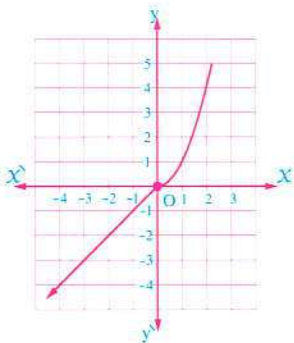
(a) $y = 0$ (b) $y = x$ (c) $y = -x$ (d) $x = 0$

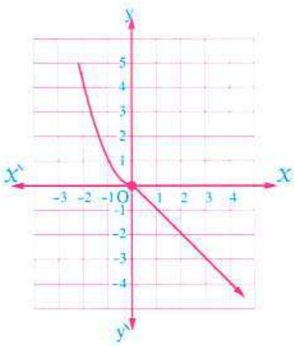
(8) The function $f : f(x) = \begin{cases} -x^2 & , \quad x < 0 \\ \frac{1}{x} & , \quad x > 0 \end{cases}$ is increasing on ....

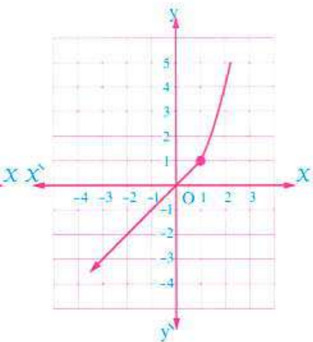
(a) $\mathbb{R}$ (b) $\mathbb{R}^-$ (c) $\mathbb{R}^+$ (d) $\mathbb{R} - \{0\}$

(9) The curve of the function $f : f(x) = \begin{cases} x^2 & , \quad x > 0 \\ x & , \quad x \le 0 \end{cases}$ is ....


(a)

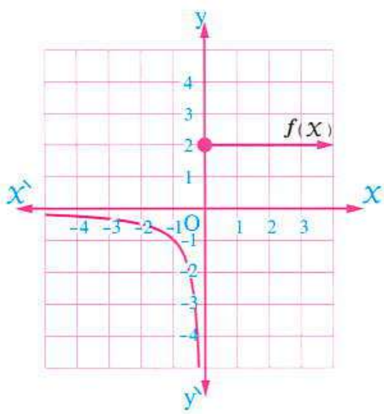

(b)


(c)


(d)

(10) In the opposite figure :

The curve of the function f is defined by the rule $f(x) =$ ....



(a) $\begin{cases} 2 & , \quad x > 0 \\ \frac{1}{x} & , \quad x < 0 \end{cases}$

(b) $\begin{cases} 2 & , \quad x \ge 0 \\ \frac{1}{x} & , \quad x < 0 \end{cases}$

(c) $\begin{cases} 2 & , \quad x < 0 \\ \frac{1}{x} & , \quad x > 0 \end{cases}$

(d) $\begin{cases} 2 & , \quad x \ge 2 \\ \frac{1}{x} & , \quad x < 2 \end{cases}$

Second Essay questions

1 **Graph each of the following functions and determine its range :**

(1) $f : \{-3 , -1 , 1 , 2\} \longrightarrow [-3 , 7]$, $f(x) = 2x + 3$

(2) $g : [1 , 5[\longrightarrow \mathbb{R}$, $g(x) = x + 1$

(3) $g :]-\infty , -1[\longrightarrow \mathbb{R}$, $g(x) = 1 - x$

(4) $f : f(x) = -3x + 7$ for every $x \in \mathbb{R}$

2 If $f : [-2 , 6] \longrightarrow \mathbb{R}$ where $f(x) = \begin{cases} 4 - x & , -2 \le x < 1 \\ x & , 1 \le x \le 6 \end{cases}$

, graph the function f and from the graph deduce its range and discuss its monotonicity.

3 **Graph each of the functions defined by the following rules and from the graph , find the domain and the range of each function and discuss its monotonicity and its type whether the function is even , odd or otherwise showing its symmetry :**

(1) $f(x) = \frac{3x^2 - 3}{x^2 - 1}$

(2) $g(x) = \frac{4 - x^2}{x + 2}$

4 **Represent graphically each of the functions that are defined by the following rules , from the graph find the domain and the range of each function and discuss its monotonicity and its type whether it is even , odd or otherwise and show its symmetry :**

(1) $f :]-\infty , 3[\longrightarrow \mathbb{R}$ where $f(x) = 2$

(2) $f(x) = \begin{cases} 2 & , x \le 0 \\ -3 & , x > 0 \end{cases}$

(3) $f(x) = \begin{cases} 2 & , x > 1 \\ x - 2 & , x \le 1 \end{cases}$

(4) $f(x) = \begin{cases} x + 2 & , x \in [-2 , 1] \\ -x + 4 & , x \in]1 , 4] \end{cases}$

(5) $f(x) = \begin{cases} 4 & , x < -2 \\ x^2 & , x \ge -2 \end{cases}$

(6) $f(x) = \begin{cases} x^2 & , x < 0 \\ x & , x \ge 0 \end{cases}$

(7) $f(x) = \begin{cases} x^3 & , x < 1 \\ 1 & , x > 1 \end{cases}$

(8) $f(x) = \begin{cases} x^3 & , x < 1 \\ 2 - x & , x \ge 1 \end{cases}$

(9) $f(x) = \begin{cases} |x| & , x \le 0 \\ \frac{1}{x} & , x > 0 \end{cases}$

(10) $f(x) = \begin{cases} |x| & , x \le 0 \\ x^2 & , x > 0 \end{cases}$

(11) $f(x) = \begin{cases} 3 & , x \le -3 \\ |x| & , -3 < x < 3 \\ 3 & , x \ge 3 \end{cases}$

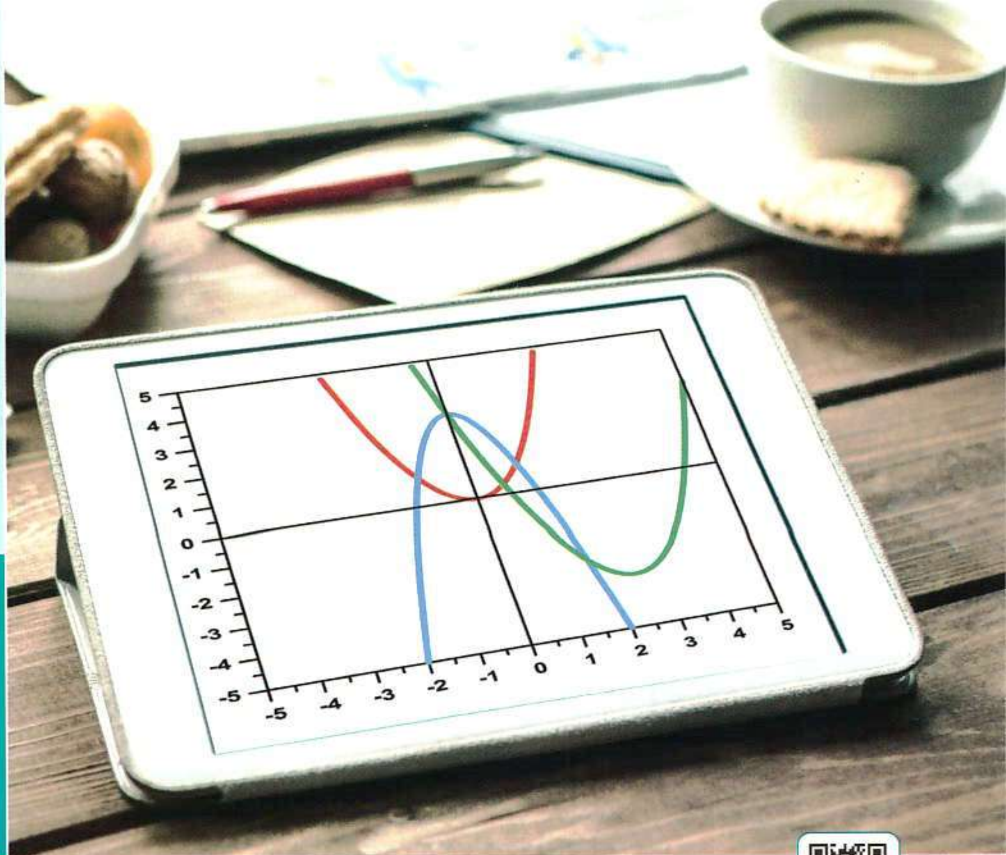
(12) $f(x) = \begin{cases} 2 & , -3 \le x \le -1 \\ 0 & , -1 < x < 1 \\ 2 & , 1 \le x \le 3 \end{cases}$

(13) $f(x) = \begin{cases} -x - 1 & , -4 \le x < -2 \\ 1 & , -2 \le x \le 2 \\ x - 1 & , 2 < x \le 4 \end{cases}$

Exercise

4

Geometrical transformations of basic function curves



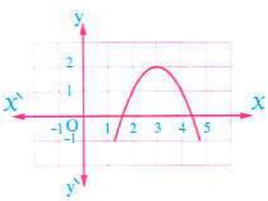
From the school book | Understand | Apply | Higher Order Thinking Skills

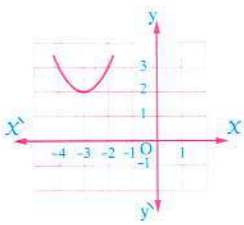
Test yourself

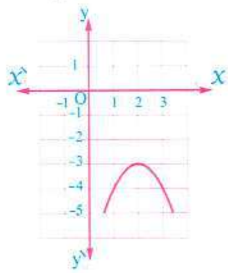
First | Multiple choice questions

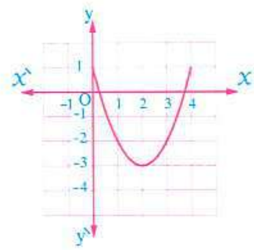
Choose the correct answer from the given ones :

(1) If $f(x) = -(x-3)^2 + 2$, then the graph that represents the function f is ……………

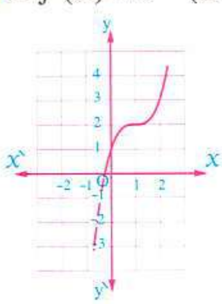

(a)

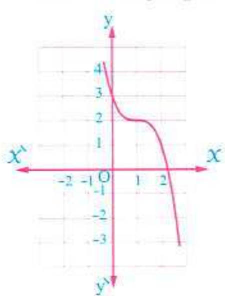

(b)

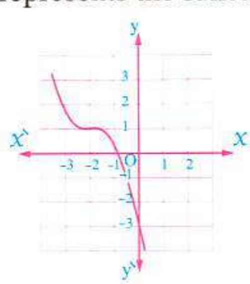

(c)

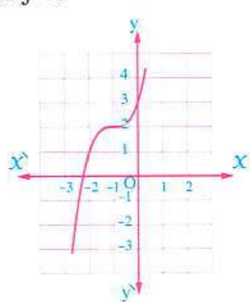

(d)

(2) If $f(x) = 2 - (x-1)^3$, then the graph that represents the function f is ……………

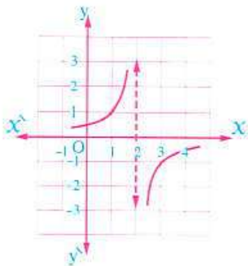

(a)

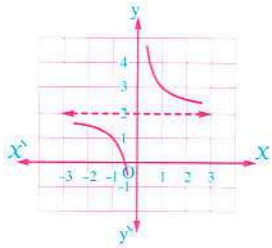

(b)

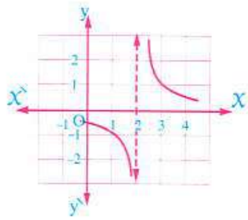

(c)

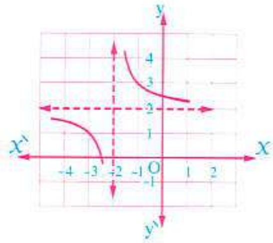

(d)

(3) If $f(X) = \frac{1}{X-2}$, then the graph that represents the function f is ....

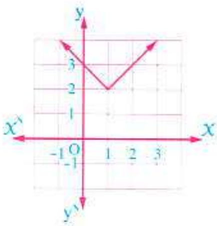

(a)

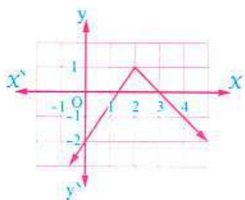

(b)

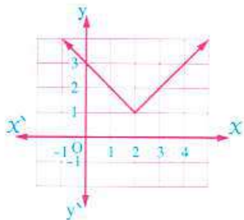

(c)

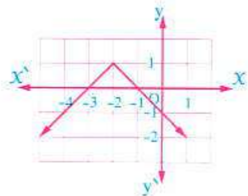

(d)

(4) If $f : f(X) = 1 - |X - 2|$, then the figure which represents the function f is ....


(a)


(b)


(c)


(d)

(5) If the curve of the function g : $g(X) = X^2$ is translated two units in the positive directions of the two axes then the function represents this translation is f : ....

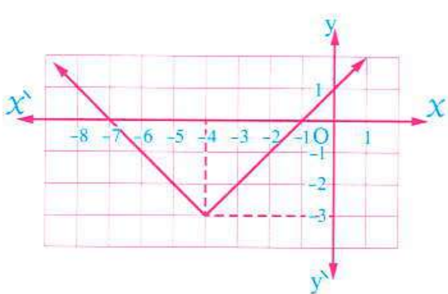
(a) $f(X) = (X+2)^2 + 2$

(b) $f(X) = (X+2)^2 - 2$

(c) $f(X) = (X-2)^2 - 2$

(d) $f(X) = (X-2)^2 + 2$

(6) Which of the following functions represents the curve in the given figure ?



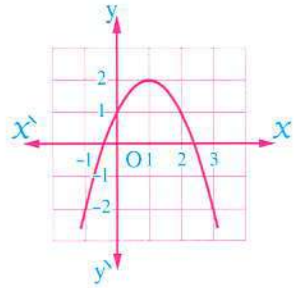
(a) $f(X) = |X - 4| - 3$

(b) $f(X) = |X - 4| + 3$

(c) $f(X) = |X + 4| - 3$

(d) $f(X) = |X + 4| + 3$

(7) Which of the following functions is represented in the given figure ?



(a) $f(X) = (X - 1)^2 + 2$

(b) $f(X) = 1 - (X - 2)^2$

(c) $f(X) = 2 - (X - 1)^2$

(d) $f(X) = (X + 1)^2 - 2$

(8) The point of the vertex of the curve of the function $f : f(X) = (2 - X)^2 + 3$ is ....

(a) (2 , 3) (b) (2 , – 3) (c) (– 2 , 3) (d) (– 2 , – 3)

(9) The symmetric point of the function $f : f(X) = X^3 - 2$ is ....

(a) (0 , 2) (b) (0 , – 2) (c) (2 , 0) (d) (– 2 , 0)

(10) The symmetric point of the function $f : f(X) = 3 - (X + 2)^2$ is ....

(a) (3 , 2) (b) (2 , 3) (c) (– 2 , 3) (d) (– 2 , – 3)

(11) The point of symmetry of the curve of the function $f : f(X) = \frac{1}{X - 3} + 4$ is ....

(a) (3 , – 4) (b) (– 3 , – 4) (c) (3 , 4) (d) (– 3 , 4)

(12) The symmetric point of the function $f : f(X) = \frac{X + 1}{X}$ is ....

(a) (1 , 0) (b) (0 , 1) (c) (0 , 0) (d) (1 , – 1)

(13) If f is a function where $f(X) = \frac{1}{X}$, then the symmetric point of the function $g : g(X) = f(X + 1)$ is ....

(a) (1 , 0) (b) (0 , 1) (c) (– 1 , 0) (d) (– 1 , 1)

(14) The vertex of the curve of the function $f : f(X) = |X + 3| - 2$ is ....

(a) (3 , 2) (b) (– 3 , – 2) (c) (– 3 , 2) (d) (3 , – 2)

(15) The curve of the function $f : f(X) = |X - 2|$ is symmetric about the straight line ....

(a) $X = 2$ (b) $X = -2$ (c) $y = 2$ (d) $y = -2$

(16) The axis of symmetry of the function $f : f(X) = X^2 - 1$ is the straight line ....

(a) $X = 1$ (b) $X = 0$ (c) $y = 1$ (d) $y = 0$

(17) If $f(X) = \frac{1}{|X|}$, then the equation of the axis of symmetry of the curve of the function f is ....

(a) $y = 0$ (b) $X = 0$ (c) $y = X$ (d) $y = -X$

(18) The function $f : f(X) = (X - 1)^2 + 2$ is increasing on the interval ....

(a) $\mathbb{R}$ (b) $]1 , \infty[$ (c) $]-\infty , 1[$ (d) $]-1 , 1[$

(19) The function f where $f(X) = \frac{2X - 1}{X - 1}$ is decreasing on the interval ....

(a) $]-\infty , 1]$ (b) $]-\infty , 1[,]1 , \infty[$

(c) $[1 , \infty[$ (d) $]-\infty , 2[,]2 , \infty[$

(20) The range of the function f where $f(X) = (X - 3)^2 + 4$ is ....

(a) $]-\infty , 3[$ (b) $[-3 , 4]$ (c) $[4 , \infty[$ (d) $]-\infty , 4]$

(21) The range of the function $f : f(X) = 3 - (2 - X)^2$ is ....

(a) $]-\infty , 2]$ (b) $[2 , \infty[$ (c) $]-\infty , 3]$ (d) $[3 , \infty[$

(22) The range of the function $f : f(X) = 2 - \frac{3}{X - 1}$ is ....

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{1\}$ (c) $\mathbb{R} - \{2\}$ (d) $\mathbb{R} - \{3\}$

(23) The range of the function $f : f(X) = |X - 2|$ is ....

(a) $]0 , \infty[$ (b) $[2 , \infty[$ (c) $[0 , \infty[$ (d) $]2 , \infty[$

(24) The range of the function $f : f(X) = 2 - |3 - 2X|$ is ....

(a) $]-\infty , 2]$ (b) $[-2 , \infty[$ (c) $]\frac{3}{2} , \infty[$ (d) $]-\infty , -2]$

(25) The range of the function $f : f(X) = \frac{|X|}{X}$ is ....

(a) $]0 , \infty[$ (b) $]-\infty , 0[$ (c) $\mathbb{R} - \{0\}$ (d) $\{1 , -1\}$

(26) The curve of the function $f : f(X) = \frac{1}{X - 3} + 4$ does not intersect the line ....

(a) $X = -3$ (b) $X = 3$ (c) $y = -4$ (d) $y = 3$

(27) If $y = f(X)$ is a real function , then its image by translation 3 units vertically upwards is $g(X) =$ ....

(a) $f(X - 3)$ (b) $f(X + 3)$ (c) $f(X) + 3$ (d) $(X) - 3$

(28) If the curve $y = f(X)$ represents a real function then its image by translation 5 units vertically downward is the same as $g(X) =$ ....

(a) $f(X - 5)$ (b) $f(X + 5)$ (c) $f(X) + 5$ (d) $f(X) - 5$

(29) The curve of the function $g : g(X) = X^2 + 4$ is the same curve of the function $f : f(X) = X^2$ by a translation of magnitude 4 units in the direction of ....

(a) $\overrightarrow{OX}$ (b) $\overrightarrow{OX'}$ (c) $\overrightarrow{Oy}$ (d) $\overrightarrow{Oy'}$

(30) The curve of the function g where $g(X) = |X| - 2$ is the same as the curve of the function $f : f(X) = |X|$ by translation two units in direction of ....

(a) $\overrightarrow{OX}$ (b) $\overrightarrow{OX'}$ (c) $\overrightarrow{Oy}$ (d) $\overrightarrow{Oy'}$

(31) If f is a real function whose domain is $[-3, 4]$, then the domain of $g : g(X) = f(X) + 2$ is ....

(a) $[-3, 4]$ (b) $[-1, 6]$ (c) $[-5, 2]$ (d) $\mathbb{R}$

(32) The curve of the function $g : g(X) = |X + 3|$ is the same curve of the function $f : f(X) = |X|$ by a translation of magnitude 3 units in the direction of ....

(a) $\overrightarrow{OX}$ (b) $\overrightarrow{OX'}$ (c) $\overrightarrow{Oy}$ (d) $\overrightarrow{Oy'}$

(33) If $y = f(X)$ is a real function , then its image by translation 4 units to the left is $g(X) =$ ....

(a) $f(X - 4)$ (b) $f(X + 4)$ (c) $f(X) + 4$ (d) $f(X) - 4$

(34) If f is a real function whose domain is $[-2, 3]$, then the domain of $g : g(X) = f(X - 2)$ is ....

(a) $[-2, 3]$ (b) $[-4, 1]$ (c) $[0, 5]$ (d) $\mathbb{R}$

(35) If $f : f(X) = -X^2$ move 3 units to the right and 2 units down , then resulted curve is $g(X)$, then $g(4) =$ ....

(a) -3 (b) -16 (c) 16 (d) -7

(36) If the curve $f(X) = -X^3$ moves 4 units to the left and 2 units upwards to become the curve $g(X)$, then $g(-2) =$ ....

(a) -218 (b) 214 (c) 6 (d) -6

(37) The curve of the function $g : g(X) = X$ is the same as the curve of the function $f : f(X) =$ .... by reflection in the X-axis.

(a) X (b) $-X$ (c) $X + 1$ (d) $-X + 1$

(38) The product of the slopes of the two straight lines $f(X) = aX + b$ and its image by reflection in X-axis equals ....

(a) 1 (b) -1 (c) a (d) $-a^2$

(39) The curve of the function $g : g(X) = 1 - |X|$ is the same curve of the function $f : f(X) = |X|$ by reflection in X-axis , then a translation of magnitude one unit in the direction of ....

(a) $\overrightarrow{OX}$ (b) $\overrightarrow{OX'}$ (c) $\overrightarrow{Oy}$ (d) $\overrightarrow{Oy'}$

Second Essay questions

1 Use the curve of the function f where $f(X) = X^2$ to represent each of the functions that are defined by the following rules , from the graph find the domain and the range of the function and discuss its monotonicity and its type whether it is even , odd or otherwise and write its axis of symmetry :

(1) $g(X) = X^2 - 3$
(2) $g(X) = -X^2 - 4$
(3) $g(X) = 2 - X^2$
(4) $g(X) = -(X - 3)^2$
(5) $g(X) = (X + 1)^2$
(6) $g(X) = (X - 2)^2 + 1$
(7) $g(X) = (X - 1)^2 - 2$
(8) $g(X) = (X + 2)^2 - 4$
(9) $g(X) = -\frac{1}{2} X^2$
(10) $g(X) = 2 - \frac{1}{2}(X - 5)^2$
(11) $g(X) = X^2 + 4X + 4$
(12) $g(X) = X^2 + 4X + 1$

2 Use the curve of the function f where $f(X) = X^3$ to represent each of the functions that are defined by the following rules , from the graph determine its domain , range , discuss its monotonicity and its type whether it is even , odd or otherwise and write its point of symmetry :

(1) $g(X) = X^3 + 4$
(2) $g(X) = X^3 - 5$
(3) $g(X) = (X - 3)^3$
(4) $g(X) = (X + 2)^3$
(5) $g(X) = -(X - 1)^3$
(6) $g(X) = (2 - X)^3$
(7) $g(X) = (X - 1)^3 - 2$
(8) $g(X) = (X + 1)^3 - 2$
(9) $g(X) = 2 - (X - 1)^3$
(10) $g(X) = 2X^3 - 1$

3 Use the curve of the function f where $f(X) = |X|$ to represent each of the functions that are defined by the following rules and from the graph determine its domain , range and discuss its monotonicity and its type whether it is even , odd or otherwise and write the equation of its axis of symmetry if exists :

(1) $g(X) = |X| - 3$
(2) $g(X) = 2 - |X|$
(3) $g(X) = |X - 3|$
(4) $g(X) = -|X + 5|$
(5) $g(X) = |X + 2| - 1$
(6) $g(X) = |X - 2| + 3$
(7) $g(X) = |2 - X| + 1$
(8) $g(X) = 4 - |X - 2|$

(9) $g(x) = 2|x|$

(10) $g(x) = 2|x-7|+2$

(11) $g(x) = -2|x-1|$

(12) $g(x) = 5-2|x+2|$

4 **Use the curve of the function f where $f(x) = \frac{1}{x}$ to represent each of the functions that are defined by the following rules , from the graph determine its domain , range and discuss its monotonicity and its type whether it is even , odd or otherwise and write its point of symmetry :**

(1) $g(x) = \frac{1}{x} + 2$

(2) $g(x) = \frac{-1}{x} + 1$

(3) $g(x) = \frac{-1}{x+2}$

(4) $g(x) = \frac{1}{x-3}$

(5) $g(x) = \frac{1}{x-2} + 3$

(6) $g(x) = \frac{1}{x+2} + 1$

(7) $g(x) = \frac{1}{4-x} - 3$

(8) $g(x) = \frac{x-3}{x-2}$

(9) $g(x) = \frac{2x}{x+1}$

(10) $g(x) = \frac{2x-3}{x-2}$

5 **If some geometric transformations are applied on the functions f , g , h where : $f(x) = x^2$, $g(x) = x^3$, $h(x) = \frac{1}{x}$ to get the functions represented by the following figures, complete :**

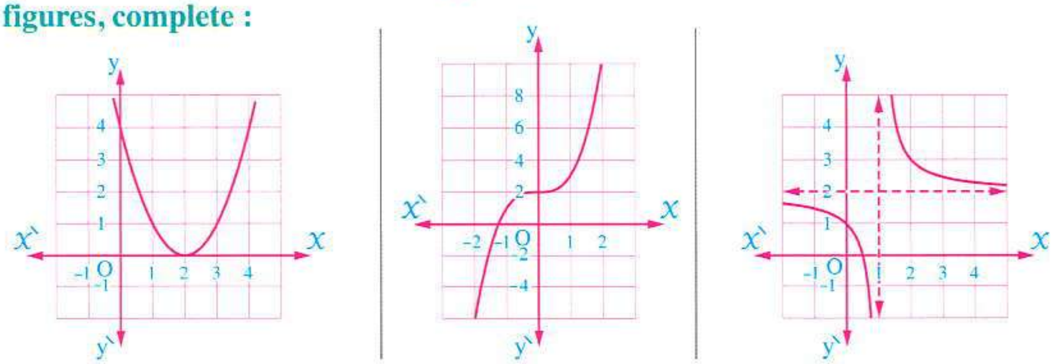


Fig. (1) Fig. (2) Fig. (3)

(1) The rule of the function in fig. (1) is ....

(2) The rule of the function in fig. (2) is ....

(3) The rule of the function in fig. (3) is ....

(4) The range of the function in fig. (1) is ....

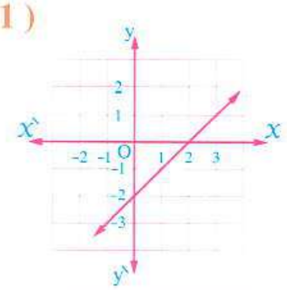
(5) The range is $\mathbb{R}$ in fig. ....

(6) The point of symmetry of the function in fig. (3) is ....

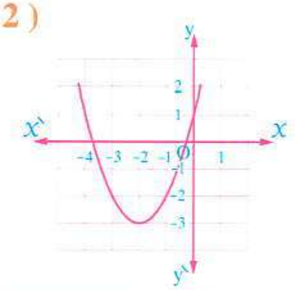
(7) The equation of symmetry line of the function in fig. (1) is ....

6 **Write the rule of the function f that is represented graphically by each of the following figures :**

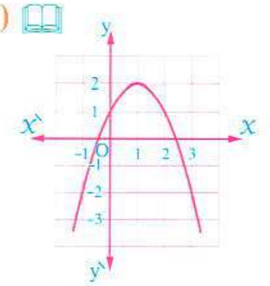
(1)



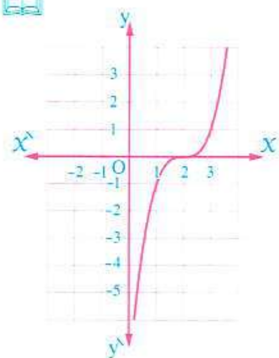
(2)



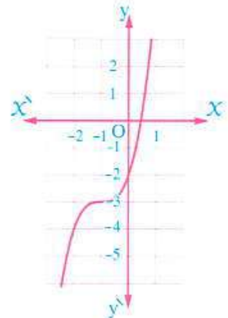
(3)



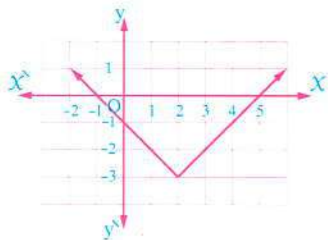
(4)



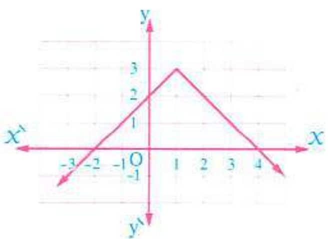
(5)



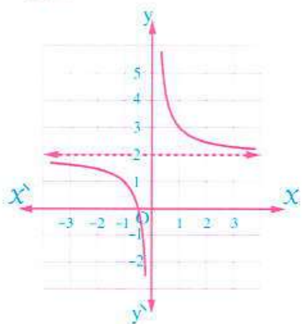
(6)



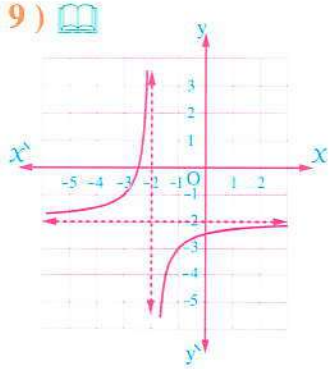
(7)



(8)



(9)



7 **If f , g , k , n are real functions where $f(x) = x^2$, $g(x) = x^3$, $k(x) = |x|$, $n(x) = \frac{1}{x}$, then represent each of the functions that are defined by the following rules showing its domain and range :**

(1) $f_1(x) = f(x+1)$

(2) $f_2(x) = f(x) - 1$

(3) $f_3(x) = 2 - f(x-1)$

(4) $g_1(x) = g(x-1)$

(5) $g_2(x) = g(x-1) + 2$

(6) $k_1(x) = \frac{1}{2}k(x) - 3$

(7) $n_1(x) = n(x-2)$

(8) $n_2(x) = 2 - n(x+1)$

8 **Draw the curve of the function f in each of the following and determine its range and discuss its monotonicity :**

(1) $f(X)=\begin{cases} X^2+1 & , \quad X>0 \\ -X^2-1 & , \quad X<0 \end{cases}$

(2) $f(X)=\begin{cases} X^2+1 & , \quad -4\le X<0 \\ -X^2-1 & , \quad 0\le X\le 4 \end{cases}$

(3) $f(X)=\begin{cases} (X-1)^3 & , \quad X\ge 0 \\ -1 & , \quad X<0 \end{cases}$

Third Higher skills

Choose the correct answer from those given :

(1) If f is a polynomial function and $f(X)=0$ at $X\in\{-3 , 1 , 0\}$, then the function $g : g(X)=f(X-3)$ cuts the X-axis at $X\in$ ....

(a) $\{-3 , 1 , 0\}$ (b) $\{3 , 0 , -2\}$ (c) $\{0 , 3 , 4\}$ (d) $\{-6 , 2 , 0\}$

(2) If $f : f(X)=(X-a+1)^2+b-2$ is a quadratic function whose range is $[1 , \infty[$ and the curve of f passes through $(3 , 2)$, then a = ...

(a) ± 4 (b) 3 or 5 (c) 3 or -5 (d) -3 or 5

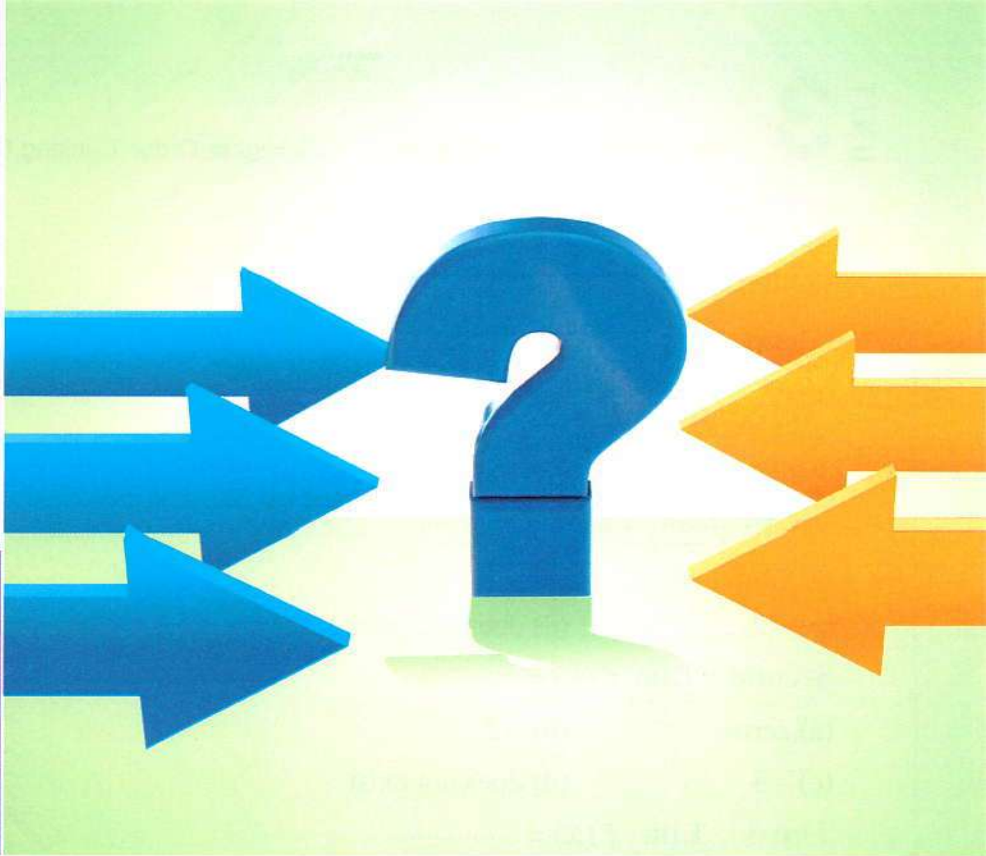
(3) The curve $y=3(X-5)^2+7$ by translation 3 units in the positive direction of X-axis and one unit in the negative direction of y-axis is ....

(a) $y=3(X+8)^2+6$ (b) $y=3(X-8)^2-6$

(c) $y=3(X-8)^2+6$ (d) $y=3(X+8)^2-6$

(4) If $f : f(X)=\begin{cases} X^3+2 & , \quad X\ge 0 \\ g(X) & , \quad X<0 \end{cases}$ is symmetric about y-axis , then g (X) = ...

(a) X^3-2 (b) X^3+2 (c) $-X^3+2$ (d) $-X^3-2$



Exercise 11

Introduction to limits of functions

"Evaluation of the limit numerically and graphically"

From the school book ● Understand ○ Apply Higher Order Thinking Skills

First Multiple choice questions

Choose the correct answer from those given :

(1) All the following are unspecified quantities except ....

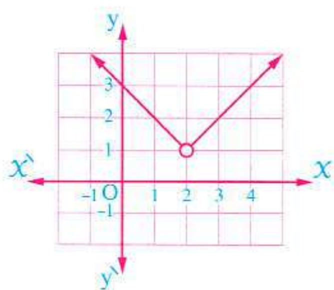
(a) zero ÷ zero (b) $\infty - \infty$ (c) $\infty + \infty$ (d) $\infty \div \infty$

(2) In the opposite figure :

$\lim\limits_{X \to 2} f(X) =$ ....

(a) 1 (b) – 1

(c) does not exist. (d) 2

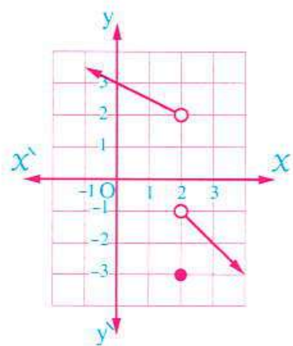


(3) In the opposite figure :

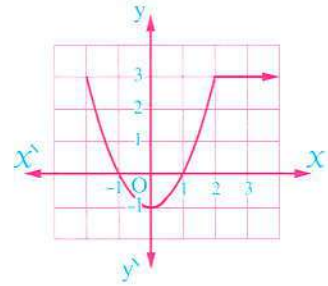
$\lim\limits_{X \to 2} f(X) =$ ....

(a) – 3 (b) 2

(c) – 1 (d) does not exist.



(4) In the opposite figure :

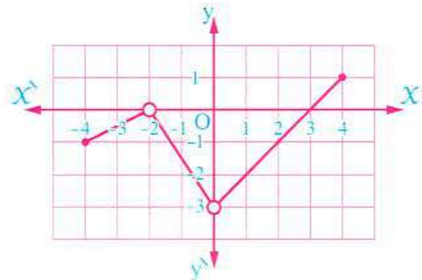


$\lim_{X \to 2} f(X) =$ ....

(a) zero (b) 2

(c) 3 (d) does not exist.

(5) From the opposite figure :



First : $\lim_{X \to -2} f(X) =$ ....

(a) zero (b) – 3

(c) – 2 (d) does not exist.

Second : $\lim_{X \to 0} f(X) =$ ....

(a) zero (b) – 2

(c) – 3 (d) does not exist.

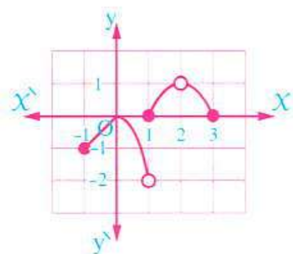
Third : $\lim_{X \to -4} f(X) =$ ....

(a) zero. (b) – 4 (c) – 1 (d) does not exist.

Fourth : $\lim_{X \to 4} f(X) =$ ....

(a) zero. (b) 4 (c) 1 (d) does not exist.

(6) Using the opposite figure :



First : $\lim_{X \to 1} f(X) =$ ....

(a) zero (b) – 2

(c) 1 (d) does not exist.

Second : $\lim_{X \to -1} f(X) =$ ....

(a) zero (b) – 1 (c) – 2 (d) does not exist.

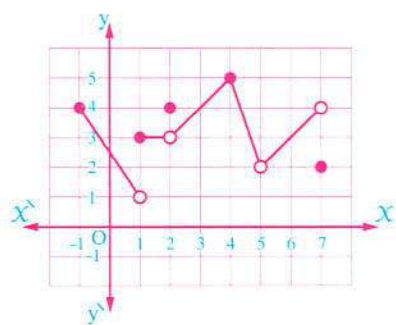
Third : $\lim_{X \to 2} f(X) =$ ....

(a) zero. (b) 1 (c) 2 (d) does not exist.

Fourth : $\lim_{X \to 0} f(X) =$ ....

(a) zero. (b) – 1 (c) – 2 (d) does not exist.

(7) Using the opposite figure :



First : $\lim_{X \to -1} f(X) =$ ....

(a) zero. (b) – 1

(c) 4 (d) does not exist.

Second : $f(2) =$ ....

(a) zero. (b) 3

(c) 4 (d) undefined.

Third : $f(5) = \dots\dots$

(a) zero. (b) 2 (c) 5 (d) undefined.

Fourth : $\lim\limits_{x \to 5} f(x) = \dots\dots$

(a) zero. (b) 2 (c) 3 (d) does not exist.

Fifth : $\lim\limits_{x \to 7} f(x) = \dots\dots$

(a) zero. (b) 2 (c) 4 (d) does not exist.

Second Essay questions

1 Complete the following table and deduce $\lim\limits_{x \to 2} f(x)$ where $f(x) = 5x + 4$:

x	1.9	1.99	1.999	⟶	2	⟵	2.001	2.01	2.1
$f(x)$	....	....	....	⟶	?	⟵	....	....	....

2 Complete the following table and deduce $\lim\limits_{x \to 2} \frac{x-2}{x^2-4}$:

x	1.9	1.99	1.999	⟶	2	⟵	2.001	2.01	2.1
$f(x)$	....	....	....	⟶	?	⟵	....	....	....

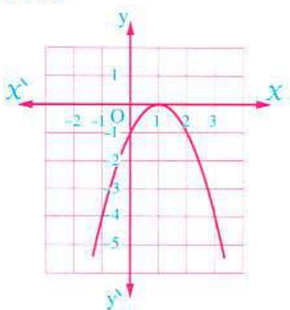
3 Find each of the following limits graphically and numerically :

(1) $\lim\limits_{x \to 4} (2x - 5)$

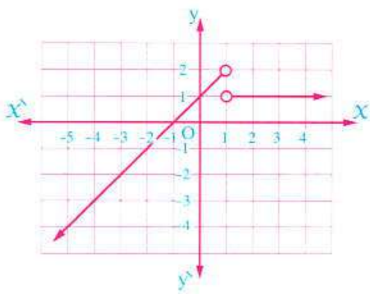
(2) $\lim\limits_{x \to 3} \frac{x^2-9}{x-3}$

4 In each of the following figures , find : $\lim\limits_{x \to 1} f(x)$

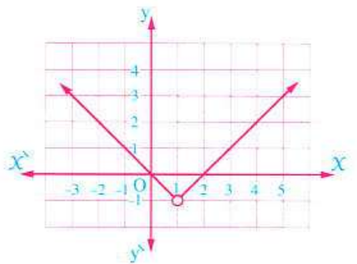
(1)



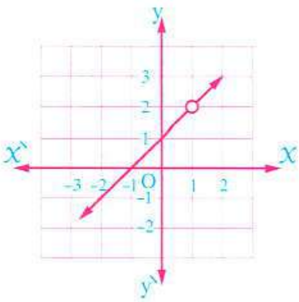
(2)



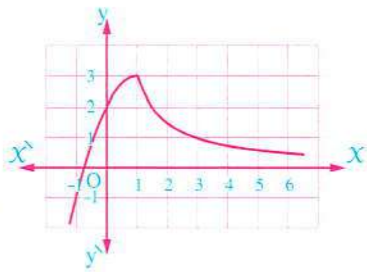
(3)



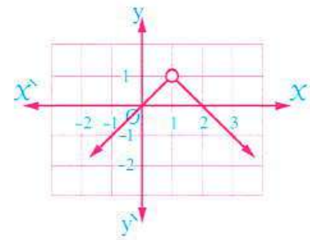
(4)



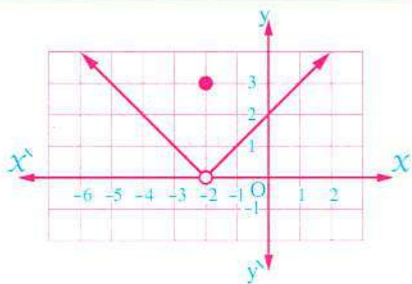
(5)



(6)



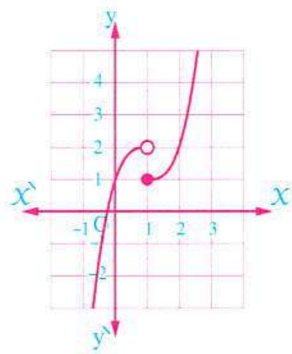
5 From the opposite figure , find :



(1) $\lim_{x \to -2} f(x)$ (2) $f(-2)$

(3) $\lim_{x \to 0} f(x)$ (4) $f(0)$

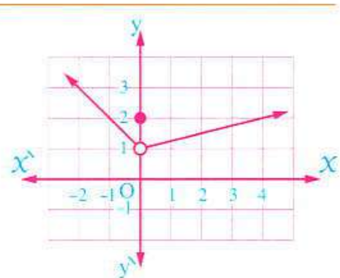
6 Study the opposite figure , then find :



(1) $f(1)$

(2) $\lim_{x \to 1} f(x)$

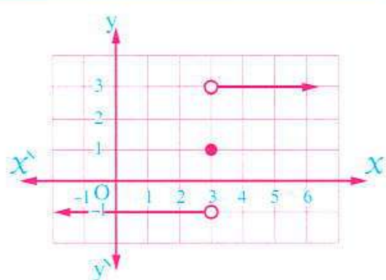
7 Study the opposite figure , then find :



(1) $f(0)$ (2) $\lim_{x \to 0} f(x)$

(3) $f(2)$ (4) $\lim_{x \to 2} f(x)$

8 From the opposite figure :



Find each of the following if possible :

(1) $f(3)$

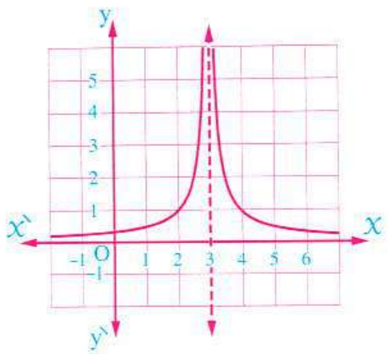
(2) $\lim_{x \to 3} f(x)$

9 **From the opposite figure :**

Find (if possible) each of the following :

(1) $f(3)$

(2) $\lim\limits_{x \to 3} f(x)$

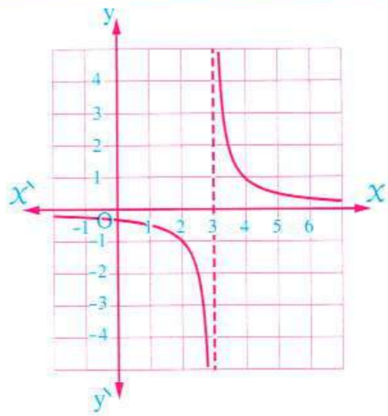


10 **From the opposite figure :**

Find (if possible) each of the following :

(1) $f(3)$

(2) $\lim\limits_{x \to 3} f(x)$



11 **From the opposite figure :**

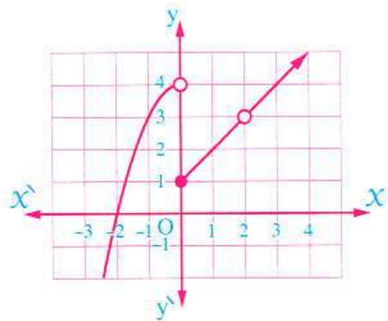
Find :

(1) $f(0)$

(2) $\lim\limits_{x \to 0} f(x)$

(3) $f(2)$

(4) $\lim\limits_{x \to 2} f(x)$



12 If the function f where $f(x) = \begin{cases} x & \text{when } x < 2 \\ x + 2 & \text{when } x \geq 2 \end{cases}$

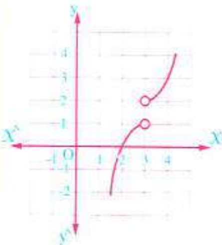
Graph the curve of this function

, then investigate graphically the presence of $\lim\limits_{x \to 2} f(x)$

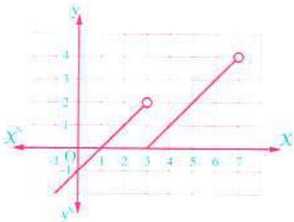
Third Higher skills

Choose the correct answer from those given :

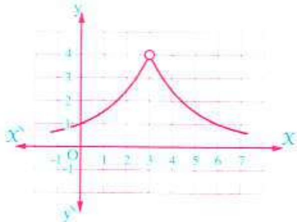
(1) Which of the functions represented by the following figures does have a limit at $X = 3$?



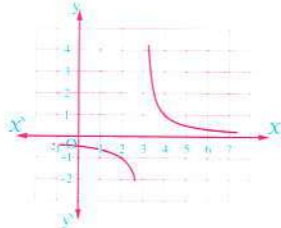
(a)



(b)

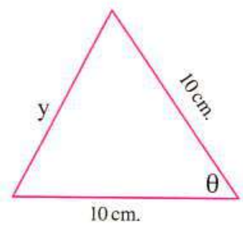


(c)



(d)

(2) In the opposite figure :



When $\theta \longrightarrow \frac{\pi}{2}$

, then : y $\longrightarrow$ .... cm.

(a) 0 (b) 5

(c) 10 (d) $10\sqrt{2}$

(3) If the curve of the polynomial function f intersects the X-axis at $X = 3$, then ....

(a) $\lim\limits_{X \to 3} f(X) = 0$ (b) $\lim\limits_{X \to 0} f(X) = 3$

(c) $\lim\limits_{X \to 0} f(X) = 0$ (d) $\lim\limits_{X \to 3} f(X) = 3$

(4) If the curve of the polynomial function f intersects the y-axis at y = 3, then ....

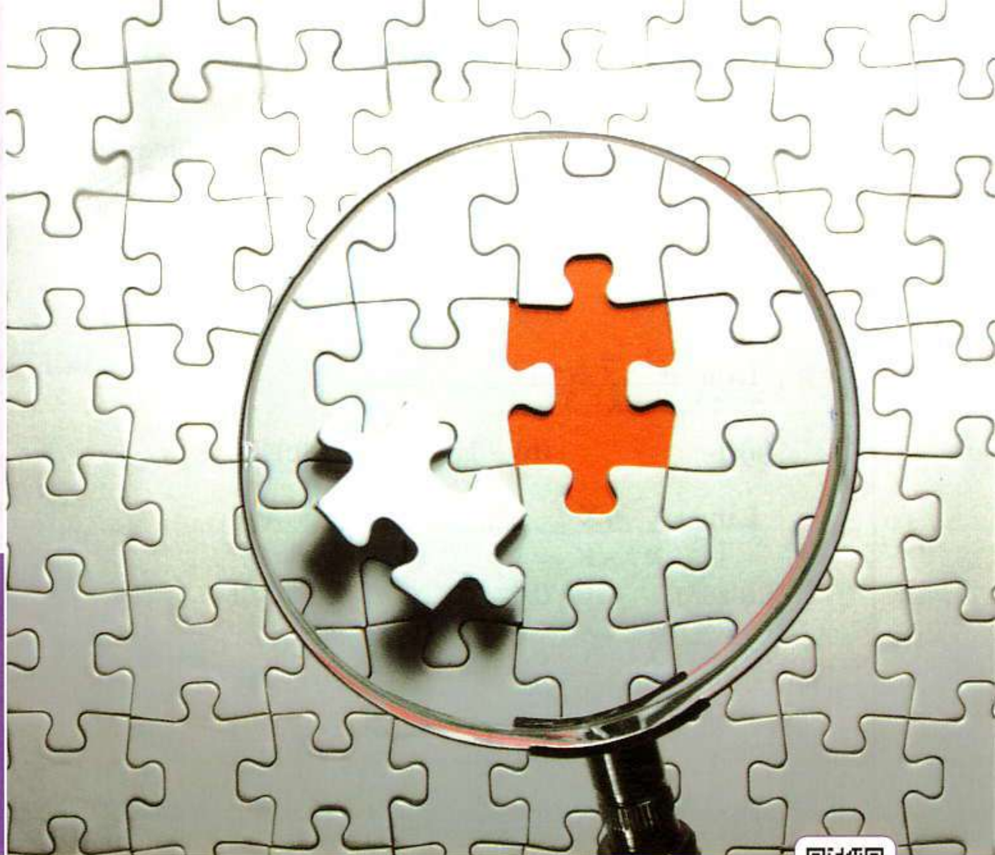
(a) $\lim\limits_{X \to 3} f(X) = \text{zero}$ (b) $\lim\limits_{X \to 3} f(X) = 3$

(c) $\lim\limits_{X \to 0} f(X) = \text{zero}$ (d) $\lim\limits_{X \to 0} f(X) = 3$

Exercise

12

Finding the limit of a function algebraically



📖 From the school book ● Understand ● Apply ● Higher Order Thinking Skills

Test yourself

First Multiple choice questions

Choose the correct answer from the given ones :

(1) $\lim\limits_{x \to \frac{1}{2}} (10) = \dots\dots\dots$

(a) 5 (b) 20 (c) 10 (d) $10\frac{1}{2}$

(2) $\lim\limits_{x \to 4} \left(3x - \sqrt{x}\right) = \dots\dots\dots$

(a) 8 (b) 10 (c) 14 (d) 16

(3) 📖 $\lim\limits_{x \to -2} \frac{1}{|x|} = \dots\dots\dots$

(a) 1 (b) – 1 (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$

(4) 📖 $\lim\limits_{x \to -2} \frac{3x^2 - 12}{x + 2} = \dots\dots\dots$

(a) 18 (b) – 3 (c) 12 (d) – 12

(5) $\lim\limits_{x \to 2} \sqrt{\frac{3 + 2x}{4x - 1}} = \dots\dots\dots$

(a) – 3 (b) 1 (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$

(6) $\lim\limits_{x \to 3} \frac{2x - 6}{7x - 21} = \dots\dots\dots$

(a) $\frac{2}{3}$ (b) $\frac{2}{7}$ (c) $\frac{3}{7}$ (d) 3

(7) $\lim_{x \to 0} \frac{x^2 - x}{x} = \ldots\ldots$

(a) zero. (b) -1 (c) does not exist. (d) 1

(8) $\lim_{x \to 3} \frac{x^2 - 7x + 12}{x - 3} = \ldots\ldots$

(a) 1 (b) -1 (c) 7 (d) -2

(9) $\lim_{x \to -1} \frac{x^2 + x}{x^3 + 1} = \ldots\ldots$

(a) zero. (b) $-\frac{1}{3}$ (c) -1 (d) has no existence.

(10) $\lim_{x \to 3} \frac{x^2 - x - 6}{x^2 + x - 12} = \ldots\ldots$

(a) $\frac{5}{7}$ (b) $\frac{1}{7}$ (c) -1 (d) -5

(11) $\lim_{x \to 2} \frac{x^2 + x - 6}{x^2 - 4} = \ldots\ldots$

(a) $\frac{4}{5}$ (b) $\frac{5}{4}$ (c) $\frac{2}{5}$ (d) $\frac{-2}{5}$

(12) $\lim_{x \to \sqrt{5}} \frac{x^4 - x^2 - 20}{x - \sqrt{5}} = \ldots\ldots$

(a) 9 (b) $2\sqrt{5}$ (c) $9\sqrt{5}$ (d) $18\sqrt{5}$

(13) $\lim_{x \to 4} \frac{(x - 3)^2 - 1}{x - 4} = \ldots\ldots$

(a) zero (b) 2 (c) 3 (d) 4

(14) 📖 $\lim_{x \to 0} \frac{\sqrt{x + 1} - 1}{x} = \ldots\ldots$

(a) zero (b) $\sqrt{2}$ (c) $\frac{1}{2}$ (d) has no existence.

(15) $\lim_{x \to 9} \frac{\sqrt{2} - \sqrt{x - 7}}{x - 9} = \ldots\ldots$

(a) $2\sqrt{2}$ (b) $\frac{\sqrt{2}}{4}$ (c) $\frac{-\sqrt{2}}{4}$ (d) $-2\sqrt{2}$

(16) $\lim_{x \to 2} \frac{(x - 3)^2 - 1}{\sqrt{x + 2} - 2} = \ldots\ldots$

(a) -6 (b) -8 (c) -2 (d) does not exist.

(17) $\lim_{x \to 1} \frac{\sqrt{2x - 1} - 1}{\sqrt{3x + 1} - 2} = \ldots\ldots$

(a) 1 (b) $\frac{2}{3}$ (c) $\frac{1}{2}$ (d) $\frac{4}{3}$

(18) $\lim_{x \to 1} \left(\frac{x^3}{x - 1} - \frac{1}{x - 1}\right) = \ldots\ldots$

(a) zero. (b) -3 (c) 3 (d) does not exist.

(19) $\lim\limits_{x \to 2} \dfrac{x^3 - 7x + 6}{3x^2 - 8x + 4} = \dots\dots$

(a) $\frac{5}{4}$ (b) $\frac{3}{2}$ (c) $\frac{4}{5}$ (d) $\frac{1}{3}$

(20) $\lim\limits_{x \to 0} \dfrac{7 + 2x}{\cos x} = \dots\dots$

(a) 7 (b) 8 (c) 9 (d) 1

(21) $\lim\limits_{x \to \frac{\pi}{4}} \dfrac{\tan x}{x} = \dots\dots$

(a) 0 (b) 1 (c) $\frac{4}{\pi}$ (d) does not exist.

(22) $\lim\limits_{x \to \pi} \dfrac{\cos 2x}{x} = \dots\dots$

(a) 2 (b) 1 (c) $\frac{1}{\pi}$ (d) zero

(23) $\lim\limits_{x \to \frac{\pi}{2}} \dfrac{\sin x}{x} = \dots\dots$

(a) 1 (b) $\frac{\pi}{2}$ (c) $\frac{2}{\pi}$ (d) has no existence.

(24) If $\lim\limits_{x \to 2} \dfrac{a}{x + 1} = 4$, then a = ……

(a) 3 (b) 4 (c) $\frac{2}{3}$ (d) 12

(25) $\lim\limits_{x \to 2} \dfrac{x - 3}{x - 2} = \dots\dots$

(a) -1 (b) $\frac{-3}{2}$ (c) $\frac{3}{2}$ (d) does not exist.

(26) If $\lim\limits_{x \to m} \dfrac{2x^2 - x - 3}{4x^2 - 9} = \dfrac{5}{12}$, then m = ……

(a) $\frac{3}{2}$ (b) $\frac{-3}{2}$ (c) $\frac{2}{3}$ (d) $\frac{-2}{3}$

(27) If $\lim\limits_{x \to 1} \dfrac{x^2 + 3x - 4}{x + a} = 5$, then a = ……

(a) -1 (b) 1 (c) 0 (d) 4

(28) If $\lim\limits_{x \to 2} \dfrac{x^2 - 4a}{x - 2}$ exists , then a = ……

(a) -1 (b) 1 (c) 2 (d) 4

Second Essay questions

1 **Find each of the following :**

(1) $\lim\limits_{x \to 5} \dfrac{x^2 - 25}{x - 5}$ « 10 »

(2) $\lim\limits_{x \to 3} \dfrac{x^2 - 8x + 15}{x - 3}$ « -2 »

(3) $\lim\limits_{x \to 0} \dfrac{x^2}{3x^3 - 2x^2}$ « $-\frac{1}{2}$ »

(4) $\lim\limits_{x \to 2} \dfrac{5x - 10}{4x - 8}$ « $\frac{5}{4}$ »

(5) 📖 $\lim\limits_{x \to 4} \frac{4x^2 - 64}{x - 4}$ « 32 »

(6) $\lim\limits_{x \to 4} \frac{2x - 8}{x^2 - x - 12}$ « $\frac{2}{7}$ »

(7) 📖 $\lim\limits_{x \to -1} \frac{x^2 - 1}{x^2 + x}$ « 2 »

(8) 📖 $\lim\limits_{x \to 5} \frac{x^3 - 25x}{x - 5}$ « 50 »

(9) $\lim\limits_{x \to -3} \frac{x^2 + 4x + 3}{x^2 - 9}$ « $\frac{1}{3}$ »

(10) $\lim\limits_{x \to 2} \frac{x^2 - 2x}{x^2 - x - 2}$ « $\frac{2}{3}$ »

(11) $\lim\limits_{x \to -1} \frac{x^2 - 3x - 4}{x^2 - x - 2}$ « $\frac{5}{3}$ »

(12) $\lim\limits_{x \to \frac{1}{2}} \frac{2x^2 - 5x + 2}{2x - 1}$ « $\frac{-3}{2}$ »

(13) 📖 $\lim\limits_{x \to \frac{3}{2}} \frac{2x^2 - x - 3}{4x^2 - 9}$ « $\frac{5}{12}$ »

(14) 📖 $\lim\limits_{x \to -3} \frac{2x^2 + 5x - 3}{x^2 + x - 6}$ « $\frac{7}{5}$ »

(15) 📖 $\lim\limits_{x \to 9} \frac{9 - x}{x^2 - 81}$ « $-\frac{1}{18}$ »

2 Find each of the following :

(1) $\lim\limits_{x \to 0} \frac{(x + 2)^2 - 4}{x^2 + x}$ « 4 »

(2) 📖 $\lim\limits_{x \to 0} \frac{(2x - 1)^2 - 1}{5x}$ « $\frac{-4}{5}$ »

(3) $\lim\limits_{x \to 2} \frac{(x - 3)^2 - 1}{2x^2 - 3x - 2}$ « $\frac{-2}{5}$ »

(4) $\lim\limits_{x \to -2} \frac{(x + 5)^2 - 9}{x^2 - 4}$ « $\frac{-3}{2}$ »

(5) $\lim\limits_{x \to 2} \frac{x^4 + x^2 - 20}{x - 2}$ « 36 »

(6) $\lim\limits_{x \to 2} \frac{(x^2 - 4)^2}{x - 2}$ « zero »

(7) $\lim\limits_{x \to -2} \frac{x + 2}{x^4 - 16}$ « $-\frac{1}{32}$ »

(8) 📖 $\lim\limits_{x \to 1} \frac{x^{\frac{7}{2}} - x^{\frac{1}{2}}}{x^2 - x}$ « 3 »

(9) 📖 $\lim\limits_{x \to 1} \frac{x^3 - x^2 + 2x - 2}{x - 1}$ « 3 »

(10) 📖 $\lim\limits_{x \to -1} \frac{2x^3 - x^2 - 2x + 1}{x^3 + 1}$ « 2 »

(11) $\lim\limits_{x \to 3} \left(\frac{5}{x} + \frac{x^2 - 3x}{x - 3}\right)$ « $\frac{14}{3}$ »

(12) $\lim\limits_{x \to -1} \left(\frac{x^2}{x^2 - 1} - \frac{3x + 4}{x^2 - 1}\right)$ « $\frac{5}{2}$ »

(13) $\lim\limits_{x \to 1} \left(\frac{1}{x - 1} - \frac{3}{x^3 - 1}\right)$ « 1 »

3 Find each of the following :

(1) $\lim\limits_{x \to 4} \frac{x^3 - 15x - 4}{x - 4}$ « 33 »

(2) $\lim\limits_{x \to 4} \frac{x^4 - 21x^2 + 20x}{x - 4}$ « 108 »

(3) 📖 $\lim\limits_{x \to 2} \frac{x^3 - x^2 - 5x + 6}{x - 2}$ « 3 »

(4) 📖 $\lim\limits_{x \to -3} \frac{x^3 - 10x - 3}{x^2 + 2x - 3}$ « $-\frac{17}{4}$ »

(5) $\lim\limits_{x \to -2} \frac{2x^3 + 3x^2 + 4}{x^3 + 8}$ « 1 »

(6) $\lim\limits_{x \to -2} \frac{x^2 + 4x + 4}{x^3 + x^2 - 8x - 12}$ « $-\frac{1}{5}$ »

4 **Find each of the following :**

(1) $\lim\limits_{x \to 9} \frac{\sqrt{x}-3}{x-9}$ « $\frac{1}{6}$ »

(2) $\lim\limits_{x \to 5} \frac{\sqrt{x-1}-2}{x-5}$ « $\frac{1}{4}$ »

(3) $\lim\limits_{x \to -1} \frac{x+1}{\sqrt{x+5}-2}$ « 4 »

(4) $\lim\limits_{x \to 6} \frac{x-6}{\sqrt{x-2}-2}$ « 4 »

(5) $\lim\limits_{x \to 1} \frac{\sqrt{x+3}-2}{x-1}$ « $\frac{1}{4}$ »

(6) $\lim\limits_{x \to 3} \frac{\sqrt{4x-3}-3}{x-3}$ « $\frac{2}{3}$ »

(7) $\lim\limits_{x \to 0} \frac{\sqrt{2x+9}-3}{x^2+x}$ « $\frac{1}{3}$ »

(8) $\lim\limits_{x \to 5} \frac{x^2-5x}{\sqrt{x+4}-3}$ « 30 »

(9) $\lim\limits_{x \to 1} \frac{\sqrt{x}-1}{x^2+2x-3}$ « $\frac{1}{8}$ »

(10) $\lim\limits_{x \to 3} \frac{x^2-x-6}{\sqrt{5x-6}-3}$ « 6 »

(11) $\lim\limits_{x \to 0} \frac{\sqrt{1+x}-\sqrt{1-x}}{2x}$ « $\frac{1}{2}$ »

(12) $\lim\limits_{x \to 3} \frac{\sqrt{x+1}-2}{\sqrt{x-2}-1}$ « $\frac{1}{2}$ »

---

5 If $\lim\limits_{x \to 2} \frac{f(x)-5}{x-2} = 1$, then find : $\lim\limits_{x \to 2} f(x)$ « 5 »

---

6 If $\lim\limits_{x \to -1} \frac{x^2-(a-1)x-a}{x+1} = 4$, then find a « –5 »

Third Higher skills

Choose the correct answer from those given :

(1) If f is a function satisfying that : $x(f(x)+1) = f(x)+x^2$

, then $\lim\limits_{x \to 1} f(x) =$ ....

(a) 1 (b) 2 (c) 3 (d) zero

(2) If $\lim\limits_{x \to 1} \frac{x^2+ax+b}{x-1} = 5$, then a – b = ...

(a) – 1 (b) – 4 (c) 3 (d) 7

(3) If $\lim\limits_{x \to m} \left(2f(x)-5g(x)\right) = 10$, $\lim\limits_{x \to m} \left(f(x)-g(x)\right) = 6$

, then $\lim\limits_{x \to m} \frac{f(x)}{g(x)} =$ ....

(a) $\frac{40}{7}$ (b) $\frac{2}{7}$ (c) 10 (d) 20

Exercise

15

The sine rule

From the school book ● Understand ● Apply ● Higher Order Thinking Skills

Test yourself

First Multiple choice questions

Choose the correct answer from the given ones :

(1) In any triangle XYZ , XY : YZ = ...

(a) sin X : sin Y (b) sin Y : sin Z (c) sin Z : sin X (d) sin Z : sin Y

(2) In Δ ABC , if m (∠ A) = 30° , C = $15\sqrt{3}$ cm. , m (∠ C) = 60° , then a = ... cm.

(a) 30 (b) 45 (c) 15 (d) 60

(3) DEF is a triangle in which m (∠ D) = 80° and m (∠ E) = 60° , if f = 12 cm., then d = ... cm.

(a) $\frac{12 \sin 80°}{\sin 40°}$ (b) $\frac{12 \sin 80°}{\sin 60°}$ (c) $\frac{12 \sin 40°}{\sin 80°}$ (d) $\frac{12 \cos 80°}{\cos 40°}$

(4) In Δ ABC , if a = 4 cm. , b = 7 cm. , m (∠ C) = 120° , then the area of the triangle = ... cm^2.

(a) $7\sqrt{3}$ (b) $14\sqrt{3}$ (c) 7 (d) 14

(5) 📖 XYZ is an equilateral triangle , the length of its side is $10\sqrt{3}$ cm. , then the length of the diameter of its circumcircle is .... cm.

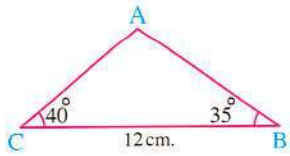
(a) 5 (b) 10 (c) 15 (d) 20

(6) In Δ XYZ , $\frac{x}{\sin X} = 6$, then the length of the diameter of its circumcircle is .... length units.

(a) 6 (b) 12 (c) 3 (d) 9

(7) In the opposite figure :

The length of $\overline{AB} \approx$ .... cm.



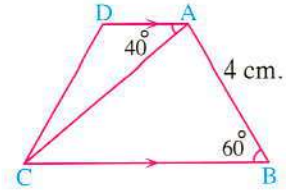
(a) 6 (b) 7

(c) 8 (d) 9

(8) In the opposite figure :

$\overline{AD} \,//\, \overline{BC}$, AB = 4 cm. , m (∠ DAC) = 40° , m (∠ B) = 60°

, then the length of $\overline{AC} \simeq$ .... cm.



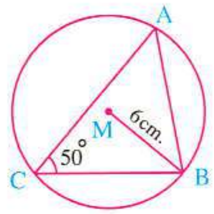
(a) 5 (b) 3

(c) 2 (d) 4

(9) In the opposite figure :

M is the centre of the circle

, BM = 6 cm. , then AB = ... cm.



(a) 6 sin 50° (b) 12 sin 50°

(c) 6 cos 50° (d) 12 cos 50°

(10) A circle with diameter of length 20 cm. , passes through the vertices of Δ ABC which is an acute-angled triangle in which BC = 10 cm. , then m (∠ A) = ...°

(a) 30 (b) 60 (c) 45 (d) 150

(11) In triangle ABC , m (∠ A) = 45° , the length of the radius of its circumcircle = 6 cm. , then a = ... cm.

(a) 13 (b) $6\sqrt{2}$ (c) 12 (d) $\sqrt{2}$

(12) If the length of a side in any triangle = 12 cm. and the measure of the opposite angle to this side = 55° , then the circumference of the circle that passes through the vertices of this triangle $\simeq$ .... cm.

(a) 36 (b) 42 (c) 46 (d) 52

(13) If the perimeter of triangle ABC equals 15 cm. , m (∠ A) = 53° , m (∠ B) = 47° , then the length of $\overline{AB} \simeq$ .... cm.

(a) 6 (b) 7 (c) 5 (d) 8

(14) In triangle ABC , a = 27 cm. , m (∠ B) = 82° , m (∠ C) = 56° , then its surface area $\simeq$ .... cm^2.

(a) 540 (b) 447 (c) 350 (d) 400

(15) In triangle ABC , m (∠ A) : m (∠ B) : m (∠ C) = 2 : 3 : 4 , AB = 12 cm. , then the length of $\overline{AC} \simeq$ .... cm.

(a) 10 (b) 11 (c) 16 (d) 18

(16) In triangle ABC , which of the following statements is true ?

(a) sin A + cos B = a + b (b) a sin B = b sin A

(c) a = b sin c (d) $\frac{a}{\sin A} = \frac{\sin B}{b}$

(17) In Δ XYZ , 2 r sin X = ... "where r is the radius length of its circumcircle"

(a) z (b) y (c) x (d) area of Δ XYZ

(18) If r is the length of the radius of the circumcircle of the triangle XYZ , then $\frac{y}{2 \sin Y}$ = ...

(a) r (b) 2 r (c) $\frac{1}{2}$ r (d) 4 r

(19) In any triangle LMN , $\frac{\ell}{\sin L}$ = ...

(a) $\frac{m}{\sin N}$ (b) $\frac{n}{\sin M}$ (c) $\frac{m + n}{\sin N + \sin M}$ (d) 3 r

(20) In Δ ABC , if $\frac{\sin A}{a} = \frac{\sin B}{b}$, then $\frac{2 \sin A - \sin B}{\text{.........}} = \frac{\sin A}{a}$

(a) a + b (b) 2 a + b (c) a – 2 b (d) 2 a – b

(21) In acute-angled triangle ABC , 2 a = $\frac{b}{\sin B}$, then m (∠ A) = ...

(a) 30° (b) 45° (c) 60° (d) 75°

(22) In Δ ABC , sin A = 2 sin C , BC = 6 cm. , then AB = ... cm.

(a) 2 (b) 3 (c) 4 (d) 6

(23) If the radius length of circumcircle of Δ ABC equals 3 cm. and sin A + sin B + sin C = 2, then the perimeter of triangle ABC = ... cm.

(a) 6 (b) 9 (c) 12 (d) 24

(24) In any triangle ABC , $\frac{\sin (A + B)}{\sin A + \sin B}$ = ...

(a) 1 (b) $\frac{c}{a + b}$ (c) $\frac{a}{b + c}$ (d) $\frac{b}{a + c}$

(25) In Δ ABC , $\frac{a}{a + b} = \frac{\sin A}{\text{.........}}$

(a) sin B (b) sin C (c) sin A + sin B (d) sin A + sin C

(26) In Δ XYZ , if 3 sin X = 4 sin Y = 2 sin Z , then x : y : z = ...

(a) 2 : 3 : 4 (b) 6 : 4 : 3 (c) 3 : 4 : 6 (d) 4 : 3 : 6

(27) ABC is a triangle in which $\frac{\sin A}{3} = \frac{2 \sin B}{5} = \frac{\sin C}{4}$, then a : b : c = ...

(a) 6 : 5 : 8 (b) 8 : 5 : 6 (c) 7 : 2 : 4 (d) 3 : 5 : 4

(28) In Δ ABC : If $\frac{\sin A}{4} = \frac{\sin B}{9} = \frac{\sin C}{7}$, then the greatest angle in measure is ....

(a) ∠ A (b) ∠ B (c) ∠ C (d) right.

(29) In triangle ABC , m (∠ A) : m (∠ B) : m (∠ C) = 3 : 5 : 4 , then $c^2 : a^2$ = ...

(a) $\sqrt{6} : 2$ (b) 2 : 3 (c) 4 : 3 (d) 3 : 2

(30) In Δ ABC , $\frac{a}{b} \times \frac{\sin B}{\sin A}$ = ...

(a) $\frac{c}{\sin C}$ (b) $\frac{\sin C}{c}$ (c) 4 r (d) 1

(31) In Δ ABC , if the radius of its circumcircle = 4 cm.

, then $\frac{a + b + c}{\sin A + \sin B + \sin C}$ = ...

(a) 4 (b) 2 (c) 8 (d) 16

(32) If the radius of the circumcircle of Δ ABC equals r , then the perimeter of the triangle = ... (sin A + sin B + sin C)

(a) r (b) 2 r (c) $4 r^2$ (d) $8 r^3$

(33) In Δ ABC , a − b = 4 cm. , sin A = $\frac{3}{2}$ sin B , then a = ... cm.

(a) 4 (b) 6 (c) 8 (d) 12

(34) If the perimeter of Δ ABC is 24 cm. and sin A + sin B = 3 sin C , then C = ... cm.

(a) 4 (b) 6 (c) 8 (d) 9

(35) ABC is a triangle , sin B + sin C = 4 sin A and b + c = 2 a + 10 cm.

, then a = ... cm.

(a) 2 (b) 3 (c) 4 (d) 5

(36) In Δ ABC , AB = 8 cm. , BC = 12 cm. , m (∠ A) − m (∠ C) = 90°

, then tan C = ...

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $\frac{3}{4}$ (d) $\frac{4}{3}$

(37) If r is the radius length of the circumcircle of Δ ABC and a = r

, then m (∠ A) = ...

(a) 30° only. (b) 30° or 120° (c) 150° only. (d) 30° or 150°

(38) If the area of the triangle ABC is Δ and r is the radius length of the circumcircle of the triangle ABC , then : $\frac{4 r \Delta}{abc}$ = ...

(a) 1 (b) 2 (c) 4 (d) 8

(39) In Δ ABC , $\frac{2 b}{\sin B}$ = ... r (where r is the radius of its circumcircle)

(a) 1 (b) 2 (c) 4 (d) 8

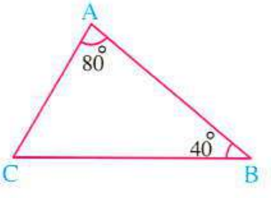
(40) If the triangle ABC is an isosceles right-angled triangle and r is the radius length of the circumcircle of the triangle ABC , then the area of Δ ABC = ... (in terms of r)

(a) $\frac{1}{2} r^2$ (b) $2 r^2$ (c) r^2 (d) $4 r^2$

(41) In the opposite figure :

If the perimeter of Δ ABC = 20 cm. ,

then the diameter length of its circumcircle ≈ .... cm.



(a) 2 (b) 4

(c) 6 (d) 8

Second Essay questions

1 XYZ is a triangle in which : m (∠ X) = 80° , m (∠ Y) = 60° and z = 10 cm.
, find each of x and y to the nearest cm. « 15 cm. , 13 cm. »

2 ABC is a triangle in which : c = 19 cm. , m (∠ A) = 112° and m (∠ B) = 33°
Find to the nearest hundredth each of b and the length of the radius of the circumcircle of the triangle. « 18.04 cm. , 16.56 cm. »

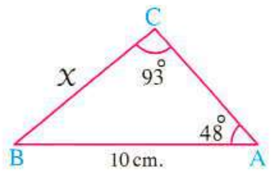
3 XYZ is a triangle , if y = 68.4 cm. , m (∠ Y) = 100° and m (∠ Z) = 40°
, find : (1) x
(2) The radius length of the circumcircle of the triangle XYZ
(3) The area of the triangle XYZ « 44.64 cm. , 34.73 cm. , 981.34 cm². »

4 ABC is a triangle in which : b = 10 cm. , m (∠ A) = 40° and m (∠ C) = 80°
Find the length of the greatest side of Δ ABC « 11 cm. »

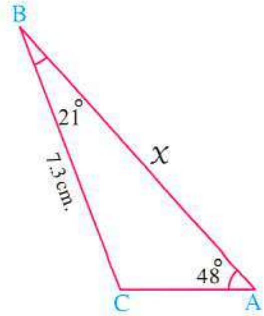
5 ABC is a triangle in which : c = 4.5 cm. , m (∠ A) = 100° and m (∠ B) = 15°
Find the length of the smallest side of Δ ABC « 1.3 cm. »

6 **Use the sine rule to find the value of x to the nearest tenth :**

(1)



(2)



« 7.4 cm. , 9.2 cm.»

7 ABC is a triangle in which : m (∠ A) = 60° and a = $7\sqrt{3}$ cm.
Find the area and the circumference of the circumcircle of Δ ABC $\left(\pi = \frac{22}{7}\right)$

« 154 cm², 44 cm. »

8 ABC is a triangle in which : a = 13 cm. , m (∠ A) = 53° 8′ , c = 15 cm. Find the radius length of the circumcircle of Δ ABC , then find m (∠ C) « 8.1 cm. , 67° 23′ 9″ or 112° 36′ 51″ »

9 ABC is a triangle in which m (∠ A) = 35° , a = 8 cm. and b = 6 cm.
Find : m (∠ B) « 25° 28′ 45″ »

10 In the triangle ABC , m (∠ A) = 67° 22′ , m (∠ C) = 44° 33′ and b = 100 cm.
Find the perimeter of the triangle ABC and its surface area. « 275 cm. , 3473 cm². »

11 ABC is a triangle in which m (∠ B) = 35° , m (∠ C) = 70° and the radius length of the circumcircle of the triangle ABC = 16 cm. Find the area and the perimeter of Δ ABC to the nearest whole number. « 262 cm², 79 cm. »

12 ABC is an isosceles triangle in which : m (∠ A) = 120° and the length of the radius of the circumcircle of Δ ABC is 12 cm.
Find c and calculate the area of Δ ABC « 12 cm. , 62.4 cm². »

13 ABC is an isosceles triangle in which : a = b and m (∠ A) = 15° and the perimeter of Δ ABC is 25 cm. Find the area of the circumcircle of Δ ABC « 474 cm². »

14 If the perimeter of Δ ABC = 40 cm. , m (∠ A) = 44° and m (∠ B) = 66°
Find the lengths of the sides of the triangle ABC « 10.9 cm. , 14.3 cm. , 14.8 cm. »

15 ABC is a triangle in which : c = 12 cm. and m (∠ B) = 3 m (∠ A) = 60°
Find a and the area of Δ ABC to the nearest cm². « 4.2 cm. , 22 cm². »

16 If the area of the triangle ABC is 450 cm², m (∠ B) = 82° and m (∠ C) = 56°
, find the value of a « 27 cm. »

17 ABC is an acute-angled triangle in which AC = 12 cm. , sin A = 0.6 and its area is 43.2 cm^2. Find the length of each of $\overline{AB}$ and $\overline{BC}$, also find m (∠ B) « 12 cm. , 7.6 cm. , 71° 34̀ »

18 Find the perimeter of the acute-angled triangle ABC if a = 7 cm. , b = 8 cm. and m (∠ A) = 60° « 20 cm. »

19 **Find the diameter length of the circumcircle of Δ ABC in the two following cases :**

(1) m (∠ A) = 75° , a = 21 cm.

(2) m (∠ B) = 50° , m (∠ C) = 65°, c – b = 6 cm. « 21.7 cm. , 42.8 cm. »

20 ABC is a triangle in which : b = 5 cm. , tan C = $\frac{4}{3}$ and m (∠ B) = 30° find a , c and the area of the triangle to the nearest integer. « 10 cm. , 8 cm. , 20 cm^2 »

21 XYZ is a triangle in which : sin X + sin Y + sin Z = 2.37 and its perimeter is 56.88 cm. Find the length of the radius of the circumcircle of Δ XYZ « 12 cm. »

22 ABC is a triangle in which sin A : sin B : sin C = 2 : 4 : 5 and c – b = 3 cm. Find each of a and b « 6 cm. , 12 cm. »

23 ABC is a triangle in which m (∠ A) : m (∠ B) : m (∠ C) = 3 : 4 : 3 , if a = 5 cm. , then find the perimeter of the triangle ABC « 15.9 cm. »

24 ABC is a triangle in which m (∠ A) = $\frac{2}{3}$ m (∠ B) = $\frac{1}{2}$ m (∠ C) , the length of the radius of its circumcircle = 10 cm. Find the area of Δ ABC « 110 cm^2 »

25 ABC is a triangle in which 6 sin A = 4 sin B = 3 sin C and its perimeter is 45 cm. Find each of a and c « 10 cm. , 20 cm. »

26 $\overline{AB}$ and $\overline{AC}$ are two chords in a circle. If their lengths are 43.5 cm. and 52.1 cm. respectively and they are drawn in two different sides of the diameter $\overline{AD}$ whose length is 100 cm.

Find : (1) m (∠ BAC) (2) The length of $\overline{BC}$

« 122° 49̀ , 84 cm. »

27 ABCD is a parallelogram in which m (∠ A) = 50° , m (∠ DBC) = 70° and BD = 8 cm. Find the perimeter of the parallelogram. « 38 cm. »

28 ABCD is a parallelogram in which AB = 18.6 cm. , m (∠ CAB) = 36° 22̀ and m (∠ DBA) = 44° 38̀

Find the length of the diagonal $\overline{AC}$ and the area of the parallelogram. « 26.46 cm. , 292 cm^2 »

29 ABCD is a parallelogram. M is the point of intersection of its two diagonals.

Let AC = 20 cm. , m (∠ AMD) = 130° and m (∠ CAB) = 85°

Find the length of $\overline{BD}$ and the area of the parallelogram ABCD « 28.2 cm. , 216 cm^2. »

30 ABCD is a trapezium in which $\overline{AD}$ // $\overline{BC}$, AD = 20 cm. , m (∠ D) = 120° ,

m (∠ B) = 62° and m (∠ ACB) = 23° 25̀

Find the length of each of $\overline{AC}$ and $\overline{BC}$ to the nearest cm. « 29 cm. , 33 cm. »

31 ABCD is a quadrilateral in which CD = 100 cm. , m (∠ BCA) = 36° ,

m (∠ BDA) = 55° , m (∠ BCD) = 85° and m (∠ CDA) = 87°

Find the lengths of $\overline{BD}$ and $\overline{AC}$ to the nearest centimetre. « 112 cm. , 144 cm. »

32 ABCD is a quadrilateral in which m (∠ ABC) = 90° , m (∠ BAD) = 80°

, AB = AD = 10 cm. , BD = BC. Calculate the area of the quadrilateral ABCD « 102 cm^2. »

33 ABCDE is a regular pentagon, whose side length is 18.26 cm.

Find the length of its diagonal $\overline{AC}$ « 29.5 cm. »

Third Higher skills

Choose the correct answer from the given ones :

(1) If the radius length of the circumcircle of the triangle ABC equals 3 cm.

, then : $\frac{abc}{\sin A \sin B \sin C}$ = ...

(a) 3 (b) 6 (c) 27 (d) 216

(2) If ABC is a triangle , then : a csc A + b csc B + c csc C = ...

(a) 2 r (b) 4 r (c) 6 r (d) 8 r

(3) If a = sin B , b = sin C , c = sin A , then the circumference of the circumcircle of Δ ABC equals .... length unit.

(a) 1 (b) $\frac{\pi}{2}$ (c) π (d) 2 π

(4) In Δ ABC , $\frac{a \sin A + b \sin B + c \sin C}{a^2 + b^2 + c^2}$ = ...

(a) $\frac{1}{r^2}$ (b) $\frac{1}{2r}$ (c) 2 r (d) r^2

FIRST Monthly tests of October

Test 1

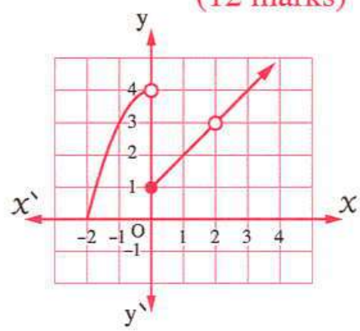
Total mark

....

20

1 Choose the correct answer from the given ones : (12 marks)

(1) In the opposite figure :



$\lim\limits_{x \to 0} f(x) =$ ....

(a) 2

(b) 4

(c) 1

(d) not exist.

(2) In Δ ABC , $\frac{\sin A}{3} = \frac{2 \sin B}{5} = \frac{\sin C}{4}$, then a : b : c = ...

(a) 6 : 5 : 8 (b) 8 : 5 : 6 (c) 7 : 2 : 4 (d) 3 : 5 : 4

(3) $\lim\limits_{x \to 3} \frac{x^2 - 9}{x^2 - 5x + 6} =$ ....

(a) 2 (b) 3 (c) 6 (d) 4

(4) The function $f : f(x) = -x^2$ is decreasing when $x \in$ ....

(a) $\mathbb{R}$ (b) $\mathbb{R}^+$ (c) $\mathbb{R}^-$ (d) $\mathbb{R}^*$

(5) In Δ ABC , if b = 5 cm. , m ($\angle$ B) = 30° , then the circumference of its circumcircle = ... cm.

(a) $50\sqrt{3}\pi$ (b) 5π (a) $10\sqrt{3}\pi$ (d) 10π

(6) The curve of the function $f(x) = x^2 - 4$ is the same as the curve of the function $g(x) = x^2$ by translation 4 units in direction of ....

(a) $\overrightarrow{ox}$ (b) $\overleftarrow{ox}$ (c) $\overleftarrow{oy}$ (d) $\overrightarrow{oy}$

(7) If the domain of the function $f : f(x) = \frac{1}{x^2 + kx + 9}$ is $\mathbb{R} - \{3\}$, then k = ...

(a) 6 (b) – 6 (c) ± 6 (d) – 36

(8) The range of the function $f : f(x) = \frac{1}{x} + 2$ is ....

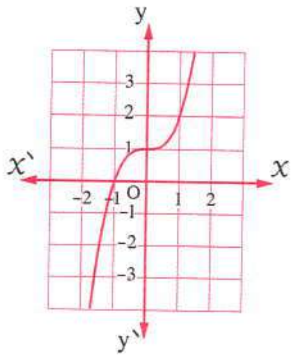
(a) $\mathbb{R} - \{-3\}$ (b) $\mathbb{R} - \{2\}$ (c) $\mathbb{R} - \{3\}$ (d) $\mathbb{R} - \{2, 3\}$

(9) The type of the function $f : f(x) = x \sin x$ is ....

(a) even. (d) odd.

(c) neither odd nor even. (d) constant.

(10) The rule of the function represented by the opposite figure is ....



(a) $y = x^3 - 1$ (b) $y = (x + 1)^3$

(c) $y = (x - 1)^3$ (d) $y = x^3 + 1$

(11) $\lim\limits_{x \to 1} \left(\frac{3}{4}\right) =$ ....

(a) 3 (b) 4 (c) $\frac{3}{4}$ (d) 1

(12) $\lim\limits_{x \to 0} \frac{\sqrt{x+1} - 1}{x} =$ ....

(a) zero (b) $\sqrt{2}$ (c) $\frac{1}{2}$ (d) not exist.

2 Answer the following questions :

(1) Graph the curve of the function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(x) = |x| - 4$, state range , type and the monotony. (2 marks)

(2) Determine the domain of the real function $f : f(x) = \frac{1}{\sqrt{3 - x}}$ (2 marks)

(3) Find : $\lim\limits_{x \to 1} \frac{\sqrt{4x - 3} - 1}{x - 1}$ (2 marks)

(4) Δ ABC in which m (∠ B) = 35° , m (∠ C) = 70° and radius of its circumcircle = 16 cm. Find area and perimeter of Δ ABC to nearest unit. (2 marks)

Test 2

Total mark .... / 20

1 Choose the correct answer from the given ones : (12 marks)

(1) The function $f : f(X) = 1 - |X|$ is increasing on ....
where $f : \mathbb{R} \longrightarrow \mathbb{R}$

(a) $]1 , \infty[$ (b) $]0 , \infty[$ (c) $]-\infty , 1[$ (d) $]-\infty , 0[$

(2) If $\lim\limits_{X \to 4} \frac{X^2 + 7X + a}{X^2 - 6X + 8} = \frac{15}{2}$, then a = ...

(a) -44 (b) 7 (c) -8 (d) 8

(3) $\lim\limits_{X \to 0} \frac{(X+5)^2 - 25}{X} =$ ....

(a) 2 (b) 25 (c) 5 (d) 10

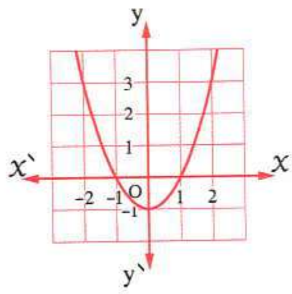
(4) The point of symmetry of the function $f : f(X) = \frac{1}{X-2} + 1$ is ....

(a) $(-2 , 1)$ (b) $(-2 , -1)$ (c) $(2 , 1)$ (d) $(2 , -1)$

(5) The domain of the function $f : f(X) = \begin{cases} 3 - X & , \quad -2 \le X < 2 \\ X & , \quad 2 \le X \le 5 \end{cases}$ is ....

(a) $[1 , 5]$ (b) $[-2 , 5]$ (c) $]1 , 5]$ (d) $[-2 , 2]$

(6) Which of the following rules defined the curve of the function shown in the opposite figure ?



(a) $f(X) = X^3 - 1$ (b) $f(X) = X^2 - 1$
(c) $f(X) = X^3 + 1$ (d) $f(X) = X^2 + 1$

(7) In Δ ABC , AB = 4 cm , sin C = $\frac{1}{3}$, then the radius of the circle passes through its vertices = ... cm.

(a) 6 (b) 8 (c) 4 (d) 12

(8) In Δ ABC , m ($\angle$ B) = 52° , m ($\angle$ C) = 48° , the perimeter of the triangle = 30 cm. , then a ≃ .... (to the nearest cm.)

(a) 15 (b) 21 (c) 12 (d) 20

(9) If f is an odd function , and its domain is $\mathbb{R}$, $a \in \mathbb{R}$, then $\frac{f(a) + f(-a)}{2} = \ldots\ldots$

(a) zero (b) $f(a)$ (c) $f(-a)$ (d) $f(0)$

(10) Which of the following rules defined a function that is not odd ?

(a) $f(x) = \sin x$ (b) $f(x) = \sec x$ (c) $f(x) = x^3$ (d) $f(x) = \frac{1}{x}$

(11) $\lim\limits_{x \to -2} \frac{1}{|x|} = \ldots\ldots$

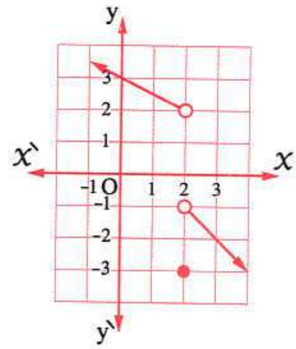
(a) 1 (b) -1 (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$

(12) **In the opposite figure :**

$\lim\limits_{x \to 2} f(x) = \ldots\ldots$

(a) -3 (b) 2

(c) -1 (d) does not exist.



2 Answer the following questions :

(1) Graph the function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(x) = (x-2)^3 - 1$, from the graph find the range , the monotony and determine if it is odd , even or otherwise. (2 marks)

(2) If f_1 , f_2 are two real functions , $f_1(x) = x^5$, $f_2(x) = \sin x$

Determine the type of the function $(f_1 + f_2)$ where it is even , odd or otherwise.

(2 marks)

(3) **Find :** $\lim\limits_{x \to -1} \frac{x+1}{\sqrt{x+5} - 2}$ (2 marks)

(4) Δ ABC in which b = 10 cm. , m ($\angle$ A) = 40° , m ($\angle$ C) = 80°

, find the length of greatest side in the triangle (2 marks)

Answers of October tests

Answers of Test 1

1

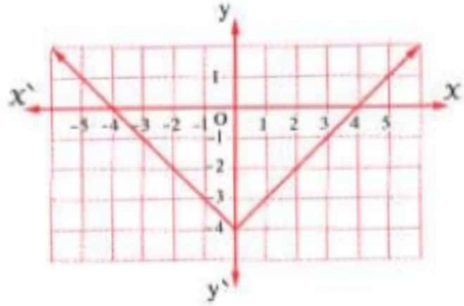
(1) d (2) a (3) c (4) b

(5) d (6) c (7) b (8) b

(9) a (10) d (11) c (12) c

2

(1)



- The range = $[-4, \infty[$
- The function is even
- The function is decreasing on $]-\infty, 0[$ and increasing on $]0, \infty[$

(2) Put $3 - x > 0$ $\quad \therefore x < 3$

$\therefore x \in]-\infty, 3[$ $\quad \therefore$ domain of $f =]-\infty, 3[$

(3) $\lim\limits_{x \to 1} \frac{\sqrt{4x-3}-1}{x-1} \times \frac{\sqrt{4x-3}+1}{\sqrt{4x-3}+1}$

$= \lim\limits_{x \to 1} \frac{4x-3-1}{(x-1)(\sqrt{4x-3}+1)}$

$= \lim\limits_{x \to 1} \frac{4(x-1)}{(x-1)(\sqrt{4x-3}+1)}$

$= \lim\limits_{x \to 1} \frac{4}{\sqrt{4x-3}+1} = \frac{4}{1+1} = 2$

(4) $\because m(\angle A) = 180° - (35° + 70°) = 75°$

$\therefore \frac{a}{\sin 75°} = \frac{b}{\sin 35°} = \frac{c}{\sin 70°} = 32$

$\therefore a = 32 \sin 75° \simeq 30.9$ cm.

, $b = 32 \sin 35° \simeq 18.4$ cm.

, $c = 32 \sin 70° \simeq 30$ cm.

$\therefore$ Area of the triangle $= \frac{1}{2} \times 30.9 \times 18.4 \times \sin 70°$

$\simeq 267$ cm^2

, perimeter of the triangle $= 30.9 + 18.4 + 30$

$\simeq 79$ cm.

Answers of Test 2

1

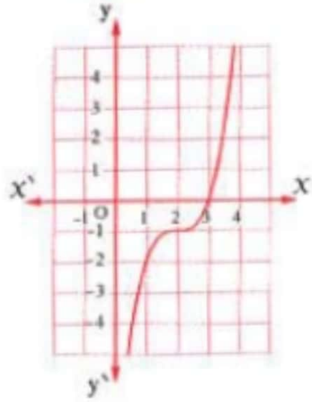
(1) d (2) a (3) d (4) c

(5) b (6) b (7) a (8) c

(9) a (10) b (11) c (12) d

2

(1)
- The range = $\mathbb{R}$
- Increasing on $\mathbb{R}$
- The function neither odd nor even.



(2) $\because f_1(-x) = (-x)^5 = -x^5 = -f_1(x)$

$\therefore f_1$ is odd function

, $\because f_2(-x) = \sin(-x) = -\sin x = -f_2(x)$

$\therefore f_2$ is odd function

$\therefore f_1 + f_2$ is odd function

(3) $\lim\limits_{x \to -1} \frac{x+1}{\sqrt{x+5}-2}$

$= \lim\limits_{x \to -1} \frac{(x+1)}{\sqrt{x+5}-2} \times \frac{\sqrt{x+5}+2}{\sqrt{x+5}+2}$

$= \lim\limits_{x \to -1} \frac{(x+1)(\sqrt{x+5}+2)}{x+5-4}$

$= \lim\limits_{x \to -1} (\sqrt{x+5}+2) = 4$

(4) $\because m(\angle B) = 180° - (40° + 80°) = 60°$

$\therefore \angle C$ is the greatest angle in measure

$\therefore c$ is the longest side

$\therefore \frac{c}{\sin 80°} = \frac{10}{\sin 60°}$

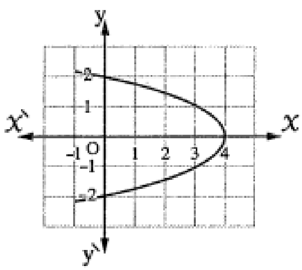
$\therefore c = \frac{10 \sin 80°}{\sin 60°} \simeq 11$ cm.

(Unit 1) Real functions

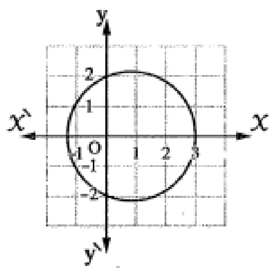
(Domain, Range and monotony)

1-

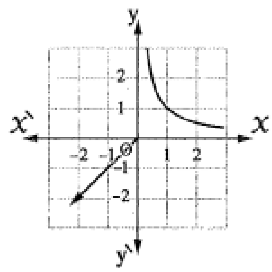
(**1**) Which of the following figures represents a function of x ?



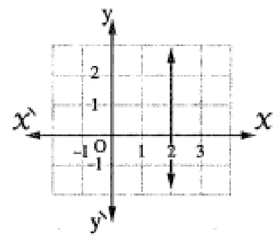
(a)



(b)

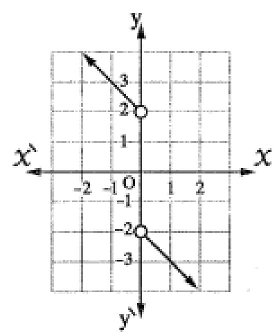


(c)



(d)

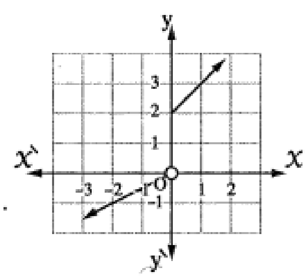
(**2**) The opposite figure represents a function of x whose domain is ....



(a) $\mathbb{R}$ (b) $\mathbb{R} -]-1 , 2[$

(c) $\mathbb{R} - [-1 , 2]$ (d) $\mathbb{R} - \{0\}$

(**3**) The opposite figure represents a function of x whose range is ....



(a) $\mathbb{R} - [0 , 2]$ (b) $\mathbb{R} - \{0\}$

(c) $\mathbb{R} - [0 , 2[$ (d) $\mathbb{R} -]0 , 2]$

(**4**) $f(x) = \sqrt{4 - x^2}$, then the domain of the function f = ...

(a) $[-2 , 2]$ (b) $]-2 , 2[$

(c) $[-2 , 2[$ (d) $]-2 , 2]$

Operations on functions (composition functions)

2-

Choose the correct answer from those given :

(**1**) $f(x) = \frac{1}{x}$, $g(x) = \sqrt{x}$, then the domain of $(f . g) =$ ....

(a) $\mathbb{R} - \{0\}$ (b) $\mathbb{R}$ (c) $\mathbb{R}^+$ (d) $[0 , \infty[$

(**2**) $f(x) = x + 1$, $g(x) = x^2$, then $(f \circ g)(2) =$ ....

(a) 3 (b) 4 (c) 5 (d) 9

(**3**) The domain of the function $f : f(x) = \sqrt{5 - x}$ equals ....

(a) $\mathbb{R} - \{5\}$ (b) $\mathbb{R}^+$ (c) $]-\infty , 5]$ (d) $[5 , \infty[$

(**4**) $f(x) = \sqrt{x}$, $g(x) = x^2$, then the domain of $(f \circ g) =$ ....

(a) $[0 , \infty[$ (b) $\mathbb{R}$ (c) $\mathbb{R}^+$ (d) $\mathbb{R}^-$

3-

If $f(x) = \frac{1}{x}$, $g(x) = x + 3$, find :

(**1**) $(f \circ g)(x)$

(**2**) $(g \circ f)(x)$

and state the domain in each case.

4-

If $f(x) = \frac{1}{x}$, $g(x) = x + 3$, find :

(**1**) $(f \circ g)(x)$

(**2**) $(g \circ f)(x)$

and state the domain in each case.

Properties of functions

5-

Find the type of each function whether it is even , odd or otherwise :

(1) $f(x) = \dfrac{x^3}{|x| + 2}$ **(2)** $f(x) = \sin x^2 - \sin^2 x$

6-

[b] If $f(x) = x - 1$, $g(x) = \sqrt{x}$, then find $(g \circ f)(x)$ and determine its domain , then find $(g \circ f)(5)$

7-

Draw the graph of the function $f : f(x) = \begin{cases} x|x| & \text{when } x < 0 \\ \dfrac{x^4}{|x|} & \text{when } x > 0 \end{cases}$

, then deduce its domain and discuss its type whether it is even , odd or otherwise.

Graphical (basic and piecewise)

8-

Graph the function $f : f(x) = 4 - (x - 2)^2$, then deduce its range , its monotony and whether the function is odd , even or otherwise.

9-

Graph the function $f : [-2 , 6] \longrightarrow \mathbb{R}$ where $f(x) = \begin{cases} 4 - x & , \quad -2 \leq x < 1 \\ x & , \quad 1 \leq x \leq 6 \end{cases}$

and from the graph deduce its range and discuss its monotonicity.

10-

Graph the function $f : f(x) = \begin{cases} x - 1 & , \quad 2 < x \leq 4 \\ -1 & , \quad -2 \leq x \leq 2 \end{cases}$

from the graph determine its range.

11-

[b] If $f : [-4 , 3] \longrightarrow \mathbb{R}$, $f(x) = \begin{cases} 4 & \text{when } x < 0 \\ (x - 1)^2 + 1 & \text{when } 0 \leq x \leq 3 \end{cases}$

, graph the function f , then from the graph , **deduce :**

(**1**) The range.

(**2**) The monotonicity.

(**3**) The type (even , odd , otherwise).

Geometric transformations

12-

Use the graph of the function f where $f(\mathcal{X}) = \mathcal{X}^2$ to represent the function g where $g(\mathcal{X}) = (\mathcal{X} - 1)^2 + 2$ and from the graph determine the range of the function g and discuss its monotonicity and tell whether it is even, odd or otherwise.

13-

Use the curve of the function f where $f(\mathcal{X}) = \mathcal{X}^3$ to represent each of the following functions :

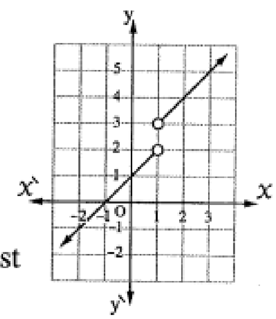
(1) $f_1(\mathcal{X}) = (\mathcal{X} + 1)^3$ (2) $f_2(\mathcal{X}) = \mathcal{X}^3 + 1$

(Unit 3) Introduction of limits of functions

1-

Choose the correct answer from those given :

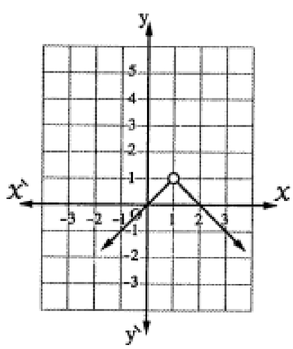
(1) The opposite figure represents the graph of the function f , then



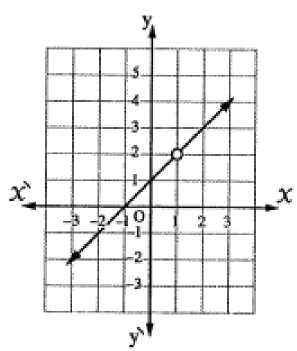
$\lim_{x \to 1} f(x) =$ ....

(a) 2 (b) 3 (c) 1 (d) not exist

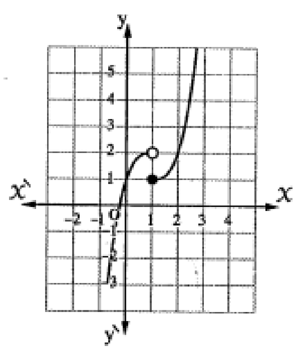
(2) Which of the following functions has no limit at $x = 1$?



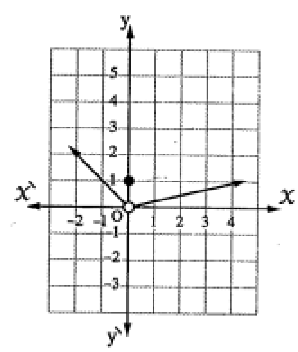
(a)



(b)



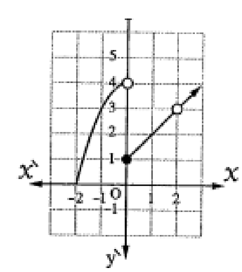
(c)



(d)

2-

In the opposite figure , find :



(1) $f(\text{zero}^+)$ **(2)** $f(\text{zero}^-)$

(3) $f(2)$ **(4)** $\lim_{x \to 2} f(x)$

Finding tke limit of the function algebrically

3-

Choose the correct answer from those given :

(1) $\lim\limits_{x \to 0} \frac{1+x}{4x-1} = \dots\dots\dots\dots$

(a) -1 (b) $\frac{1}{4}$ (c) $-\frac{1}{4}$ (d) 1

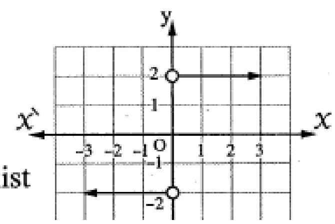
(2) $\lim\limits_{x \to 3} \frac{x^2-9}{x-3} = \dots\dots\dots\dots$

(a) -6 (b) zero (c) 3 (d) 6

(3) The opposite figure represents $f(x)$

Then : $\lim\limits_{x \to 2} f(x) = \dots\dots\dots\dots$

(a) 0 (b) -2 (c) 2 (d) not exist



4-

Find :

$$\lim_{x \to 3} \frac{x^2-x-6}{x-3}$$

5-

Find :

$$\lim_{x \to -1} \left(\frac{5x^2+5x}{3x^2-3}\right)$$

6-

$$\lim_{x \to 0} \frac{x^2+x}{x} = \dots\dots\dots$$

7-

Find : $\lim\limits_{x \to 1} \frac{x^3-2x^2+1}{x^2+x-2}$

8-

Find : (2) $\lim\limits_{x \to 2} \frac{x^2 - 5x + 6}{x^3 - 8}$

9-

Find : $\lim\limits_{x \to 5} \frac{x - 5}{2x - 3}$

10-

Find : $\lim\limits_{x \to 2} \frac{x^3 + 8}{x + 2}$

11-

$\lim\limits_{x \to -3} \frac{x^2 + 4x + 3}{x^2 - 9}$

12-

Find : (1) $\lim\limits_{x \to -2} \frac{x^3 + 8}{x + 2}$

(Unit 4) The sine rule

1-

Solve Δ ABC in which m (∠ B) = 35° , m (∠ C) = 70° , and the diameter length of its circumcircle = 32 cm.

2-

In Δ ABC , if m (∠ A) = 35° , a = 17 cm. and b = 20 cm.

Prove that : Δ ABC has two solutions , then find them.

3-

Find the perimeter of Δ ABC in which m (∠ A) = 57° 13̀ , c = 8.7 cm. and m (∠ B) = 64° 18̀

4-

ABC is a triangle in which : m (∠ A) = 35° , a = 8 cm. and b = 6 cm. **Find :** m (∠ B)

5-

ABC is a triangle in which : b = 12 cm. , m (∠ B) = 75° and m (∠ C) = 45° **Find :**

(**1**) a

(**2**) The area of Δ ABC

(**3**) The radius length of the circumcircle of the triangle ABC

6-

ABC is a triangle in which m (∠ A) : m (∠ B) : m (∠ C) = 3 : 4 : 3

If a = 5 cm. , find the perimeter of Δ ABC

7-

Solve the triangle ABC in which a = 8 cm. , m (∠ A) = 60° and m (∠ B) = 40°

8-

Solve the acute-angled triangle ABC in which a = 21 cm. , b = 25 cm. and the diameter length of its circumcircle = 28 cm.

9-

Find the shortest side length in Δ ABC , in which : m (∠ A) = 43° , m (∠ B) = 70° and c = 9 cm. Find the area of the triangle ABC.

10-

ABC is a triangle in which : AC = 4.7 cm. , m (∠ B) = 34° and m (∠ C) = 66°

Find the length of $\overline{BC}$, then find the area of its circumcircle.

11-

ABC is a triangle in which m (∠ A) = 40° , a = 5 cm. and b = 7 cm.

Find m (∠ B) approximating to the nearest minute.

12-

Solve the triangle ABC in which a = 5 cm. , b = 7 cm. and m (∠ C) = 65°

13-